Certified Non-Confluence with ConCon 1.5*

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IWC

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Conditional Term Rewriting

$$
\begin{aligned}
0 + y &\to y \\
s(x) + y &\to x + s(y) \\
f(x, y) &\to z \Leftarrow x + y \to^* z + v
\end{aligned}
$$

Conditional Term Rewriting

$$\mathsf{0} + \mathsf{y} \to \mathsf{y}$$
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$n = \mathsf{s}^n(\mathsf{0})$

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$$\mathsf{f}(1, 0) \to$$

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$$f(1, 0) \to$$
$$1 + 0$$

$$\begin{aligned} 0 + y &\to y \\ s(x) + y &\to x + s(y) \\ f(x, y) &\to z \Leftarrow x + y \to^* z + v \end{aligned}$$

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$$f(1, 0) \to$$

$$1 + 0 \to 0 + 1$$

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$n = s^n(0)$

$$
\begin{aligned}
f(1, 0) &\to 0 \\
1 + 0 &\to 0 + 1
\end{aligned}
$$

Non-Confluence in ConCon so far

Method 1

- $\ell \to r \in \mathcal{R}$
- $x \in \mathcal{V}(r) \setminus \mathcal{V}(\ell)$
- $r \in \mathsf{NF}(\mathcal{R}_{\mathsf{u}})$

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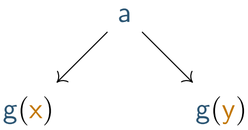
Example 1

$$\mathsf{a} \to \mathsf{g}(\mathsf{x})$$



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Method 2

- unconditional CP $s \approx t$
- $s \not\downarrow_{\mathcal{R}_\mathsf{u}} t$

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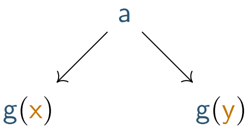

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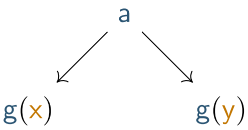
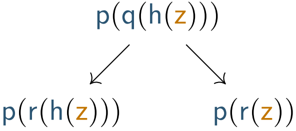


Method 2

- unconditional CP $s \approx t$
- $s \not\downarrow_{\mathcal{R}_\mathsf{u}} t$

Example 2

$$\mathsf{p}(\mathsf{q}(x)) \to \mathsf{p}(\mathsf{r}(x))$$
$$\mathsf{q}(\mathsf{h}(x)) \to \mathsf{r}(x)$$



Conditional Narrowing

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Non-Confluence using Conditional Narrowing

- $u \rightsquigarrow_{\sigma}^{*} s$
- $v \rightsquigarrow_{\tau}^{*} t$
- mgu μ s.t. $u\sigma\mu = v\tau\mu$
- $s\sigma\mu \not\downarrow_{\mathcal{R}_{\mathsf{u}}} t\tau\mu$

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$$\mathsf{f}(\mathsf{x}, \mathsf{y})\sigma \to \mathsf{x}_1\sigma$$

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$$\mathsf{f}(\mathsf{x}_1, \mathsf{y}) \to \mathsf{x}_1$$

$$\mathsf{f}(0, \mathsf{x}_3 + \mathsf{x}_4) \to \mathsf{x}_3$$

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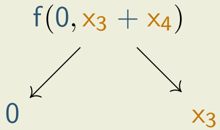
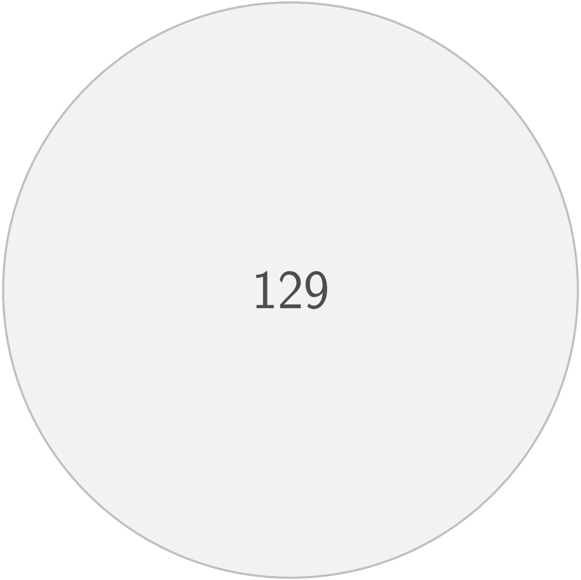


Experimental Results



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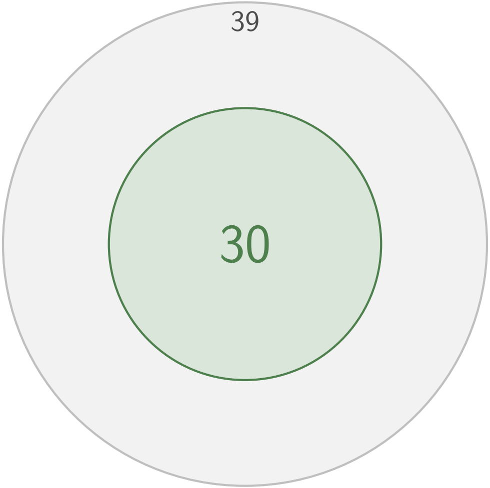

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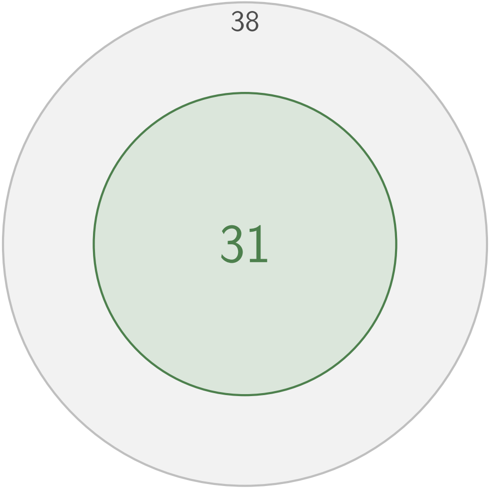

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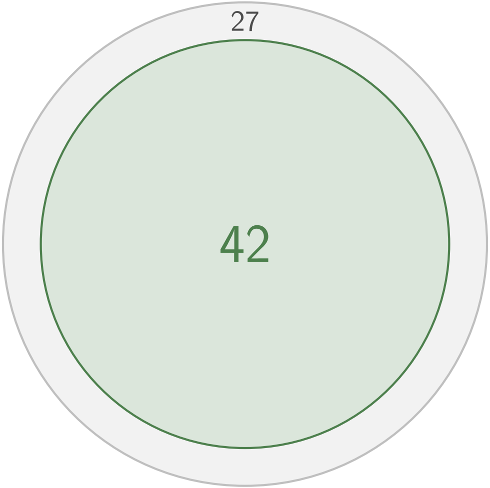

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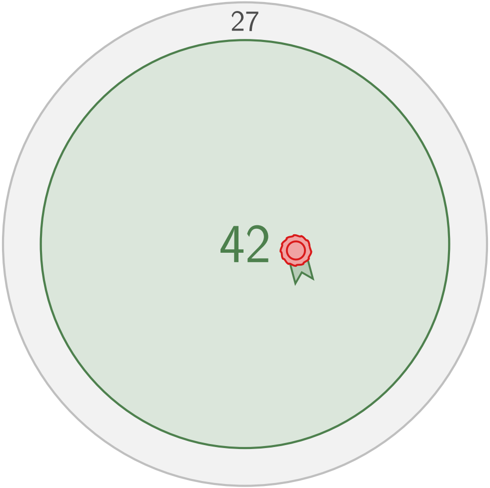

Demo

Thank you for your attention!