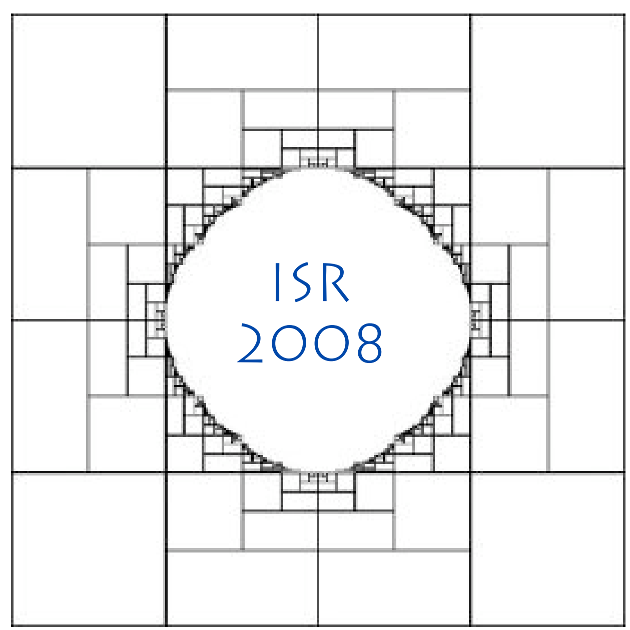
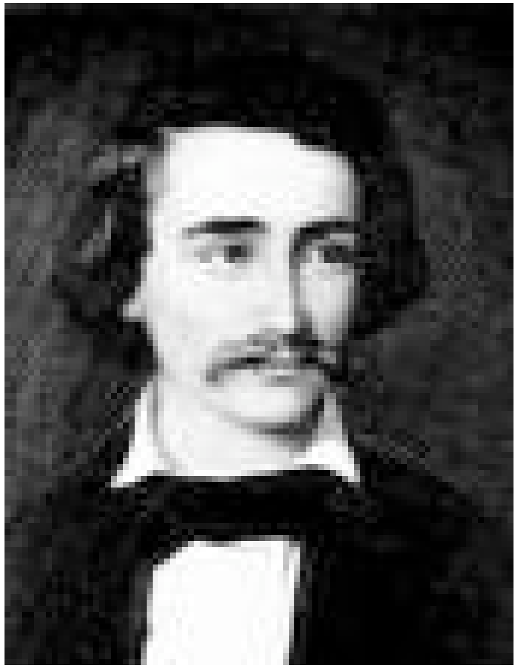
A Course in Infinitary Rewriting

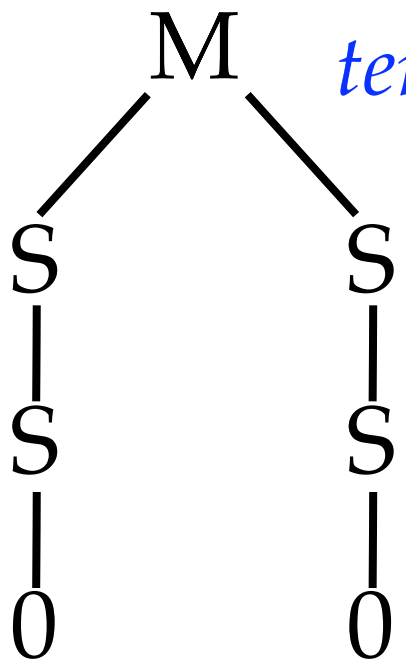
Jan Willem Klop
VU University Amsterdam



Grassmann 1861, Dedekind 1888



$$A(x, 0) \to x$$
$$A(x, S(y)) \to S(A(x, y))$$
$$M(x, 0) \to 0$$
$$M(x, S(y)) \to A(M(x, y), x)$$



termboom

data
0, S(0), S(S0)),...

footnote: Dedekind started with 1, not 0.

Therefore by (126) this sum is completely determined by the conditions*

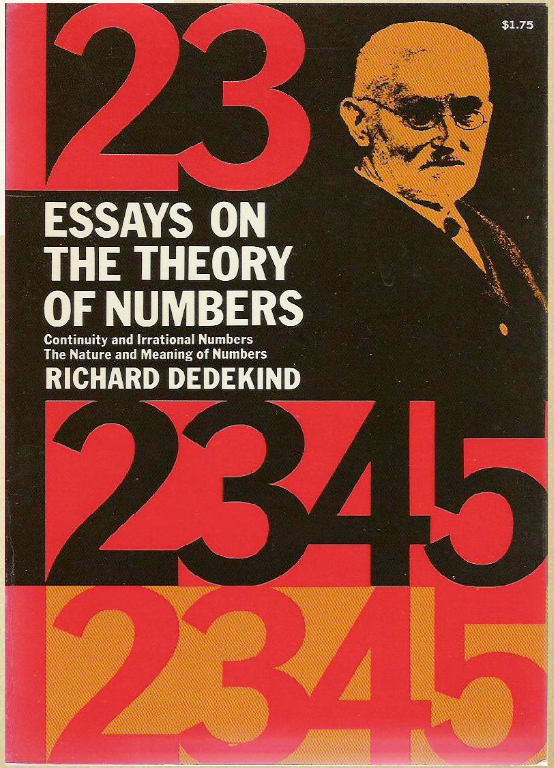
$$\text{II.}\quad m + 1 = m',$$

$$\text{1II.}\quad m + n' = (m + n)'.$$

or in short the product of the numbers m, n. This therefore by (126) is completely determined by the conditions

$$\text{II.}\quad m . 1 = m$$

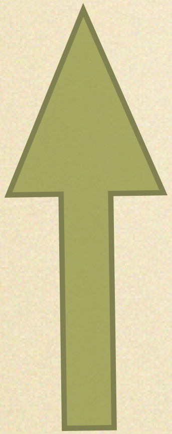
$$\text{III.}\quad m n' = m n + m,$$

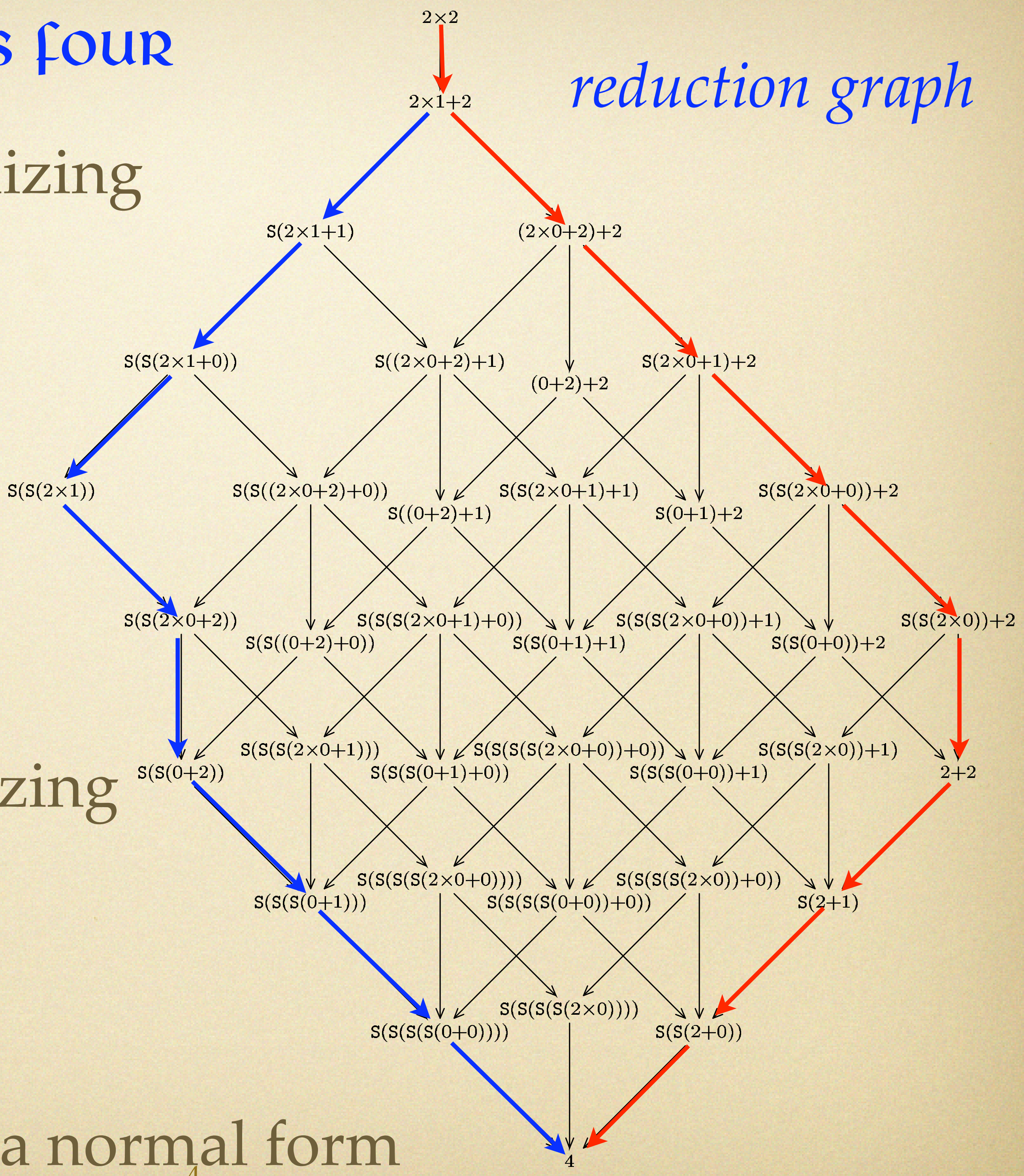


1. two times two is four

reduction graph

WN weakly normalizing



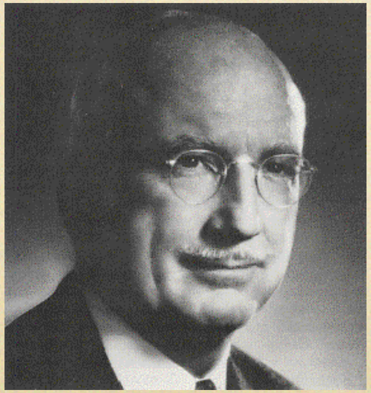


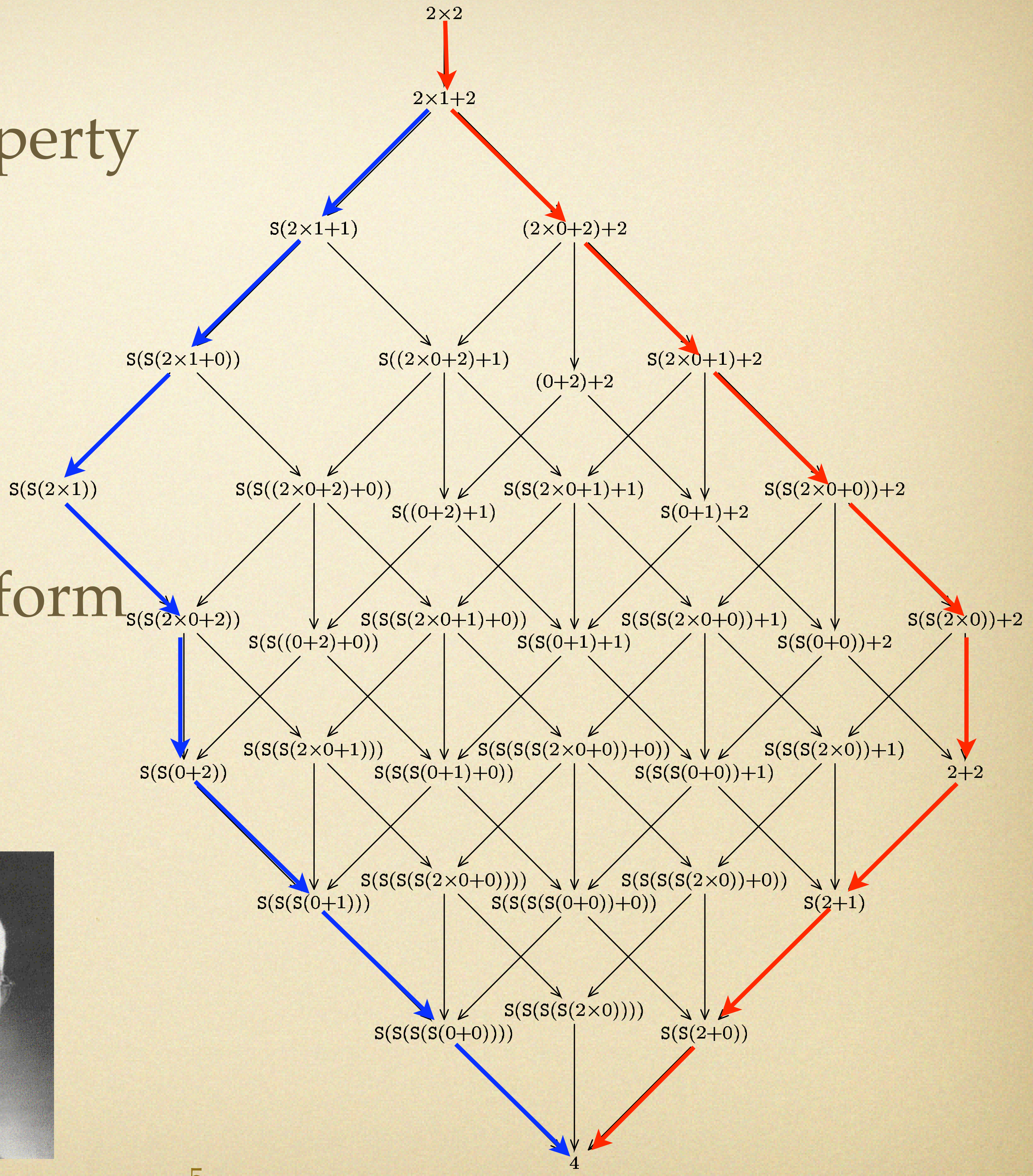
SN strongly normalizing

S(S(S(S(0)))), a normal form

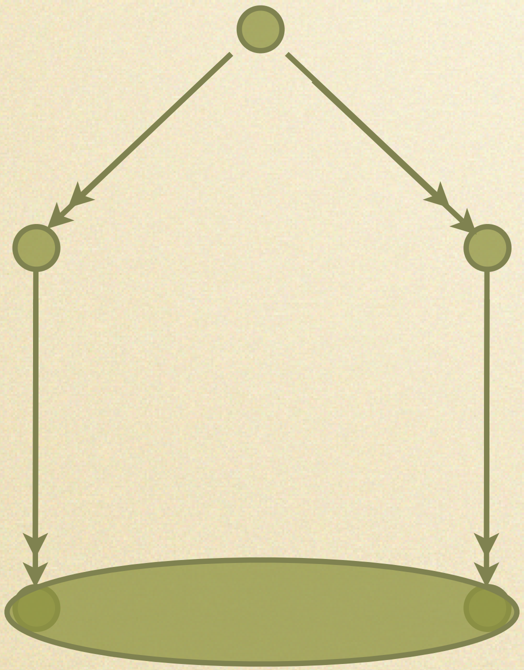
CR confluent,
Church-Rosser property

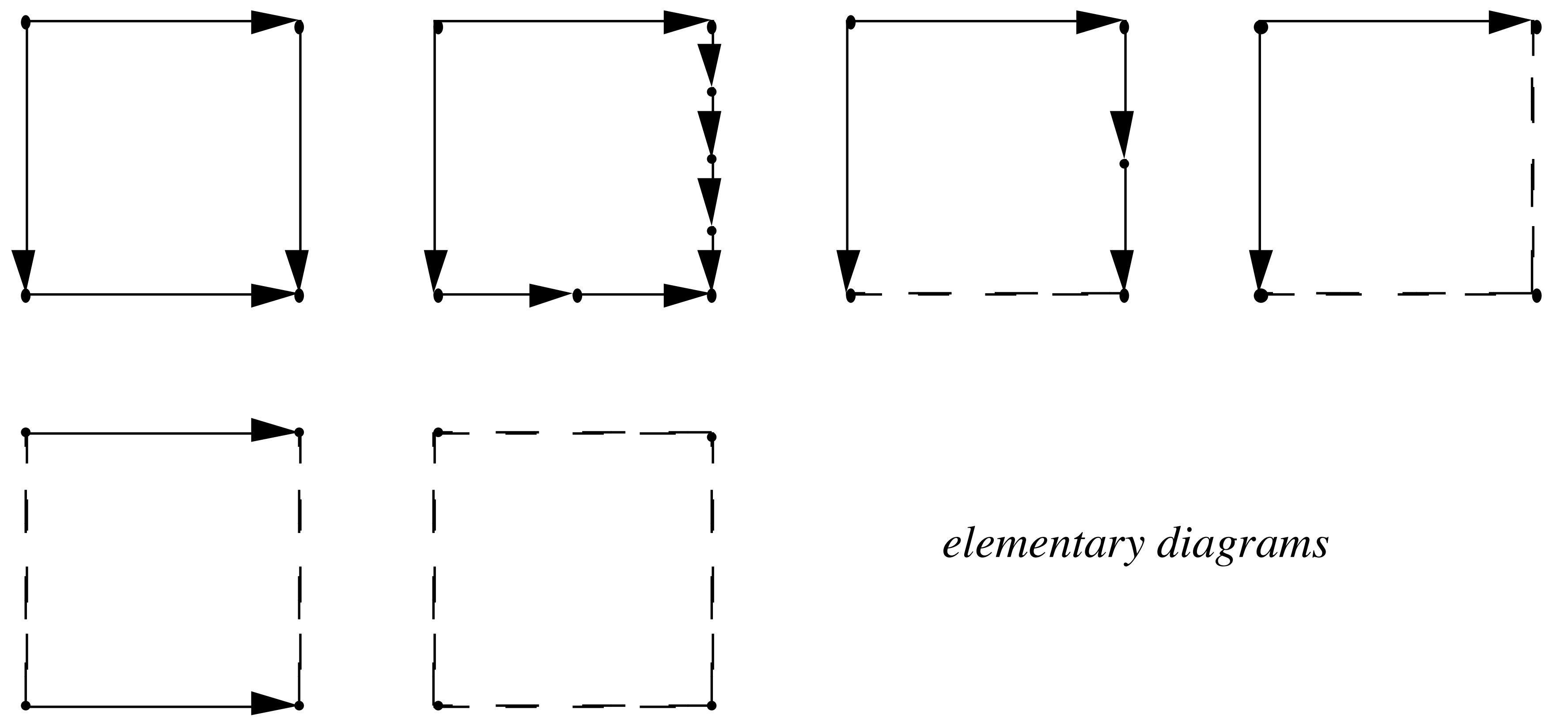
UN unique normal form property



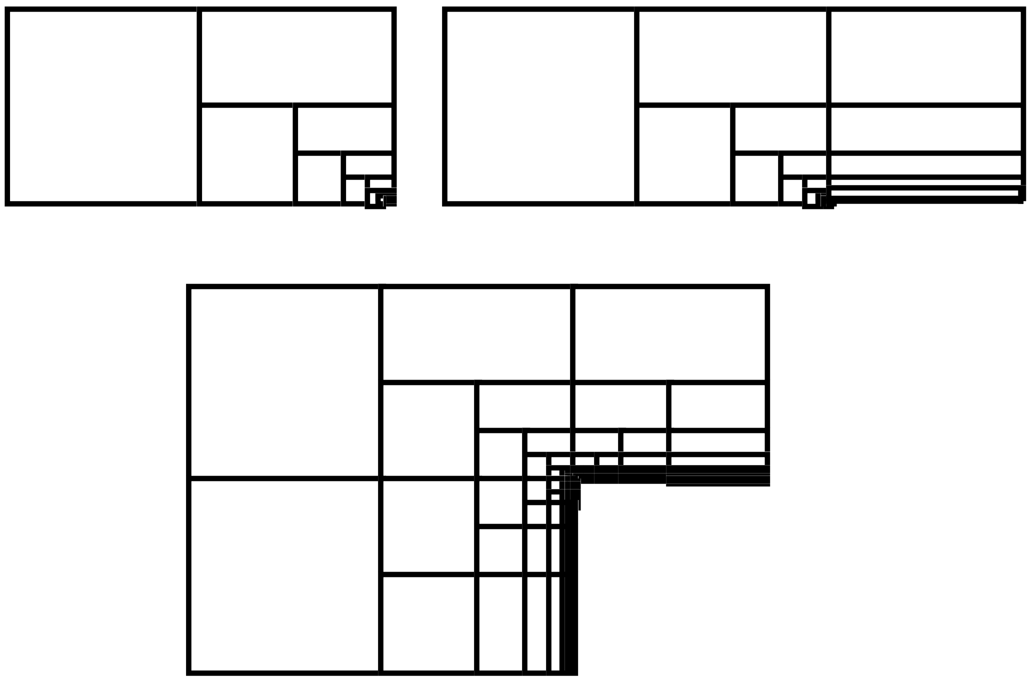


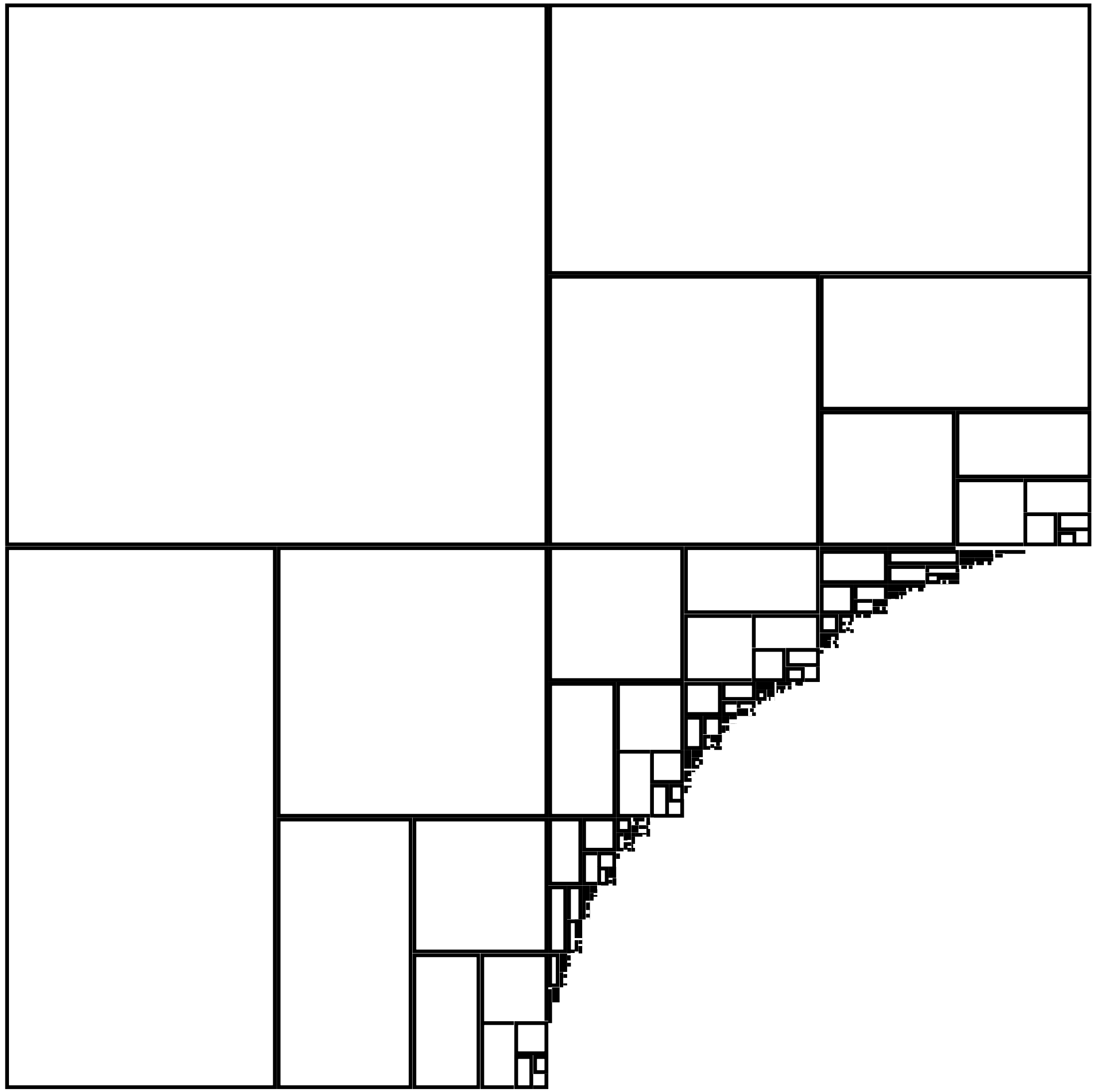
SN & UN => CR





elementary diagrams





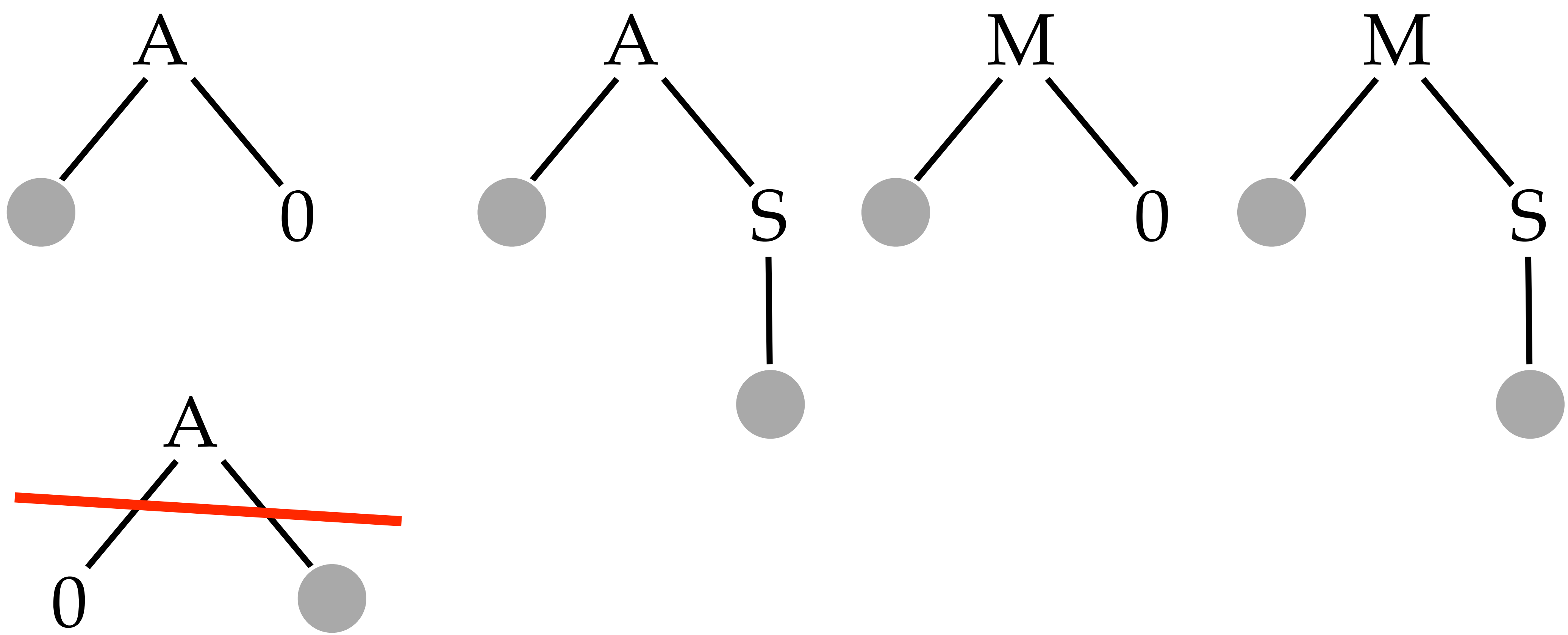
$A(x, 0) \to x$

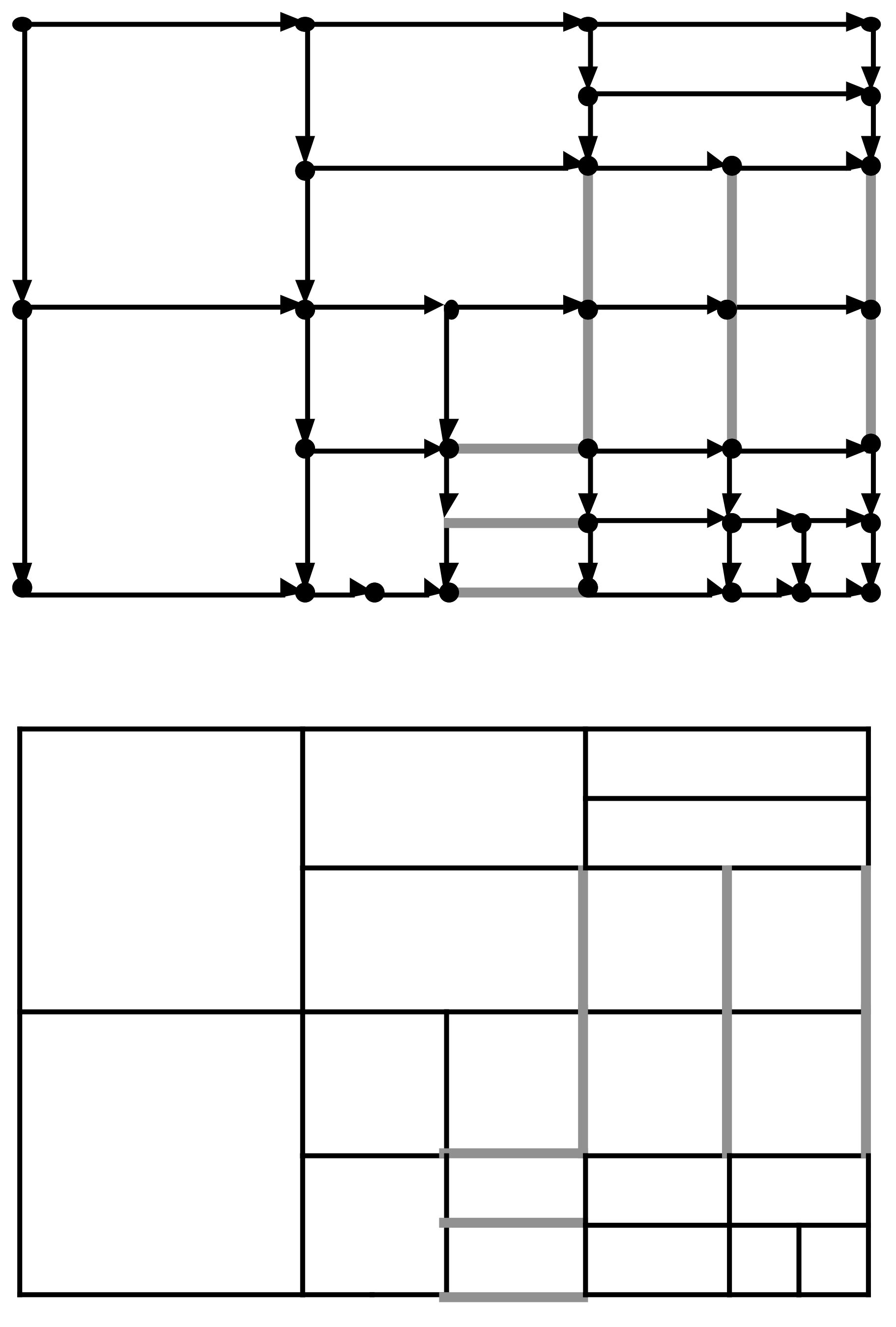
$A(x, S(y)) \to S(A(x, y))$

$M(x, 0) \to 0$

$M(x, S(y)) \to A(M(x, y), x)$

left linear
non-overlapping rules





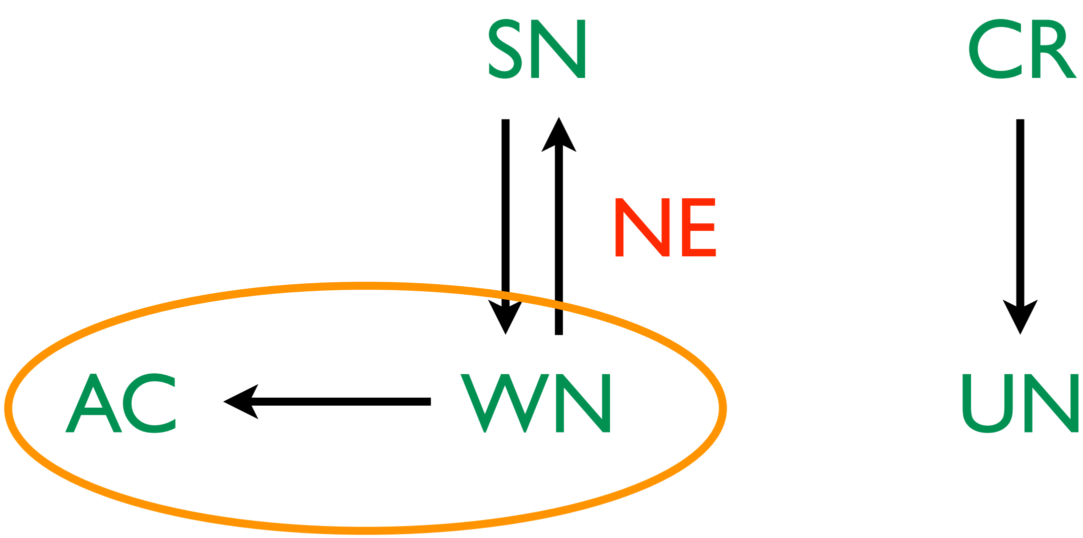
orthogonal constructor TRS
with one collapsing rule

$$A(x, 0) \to x$$

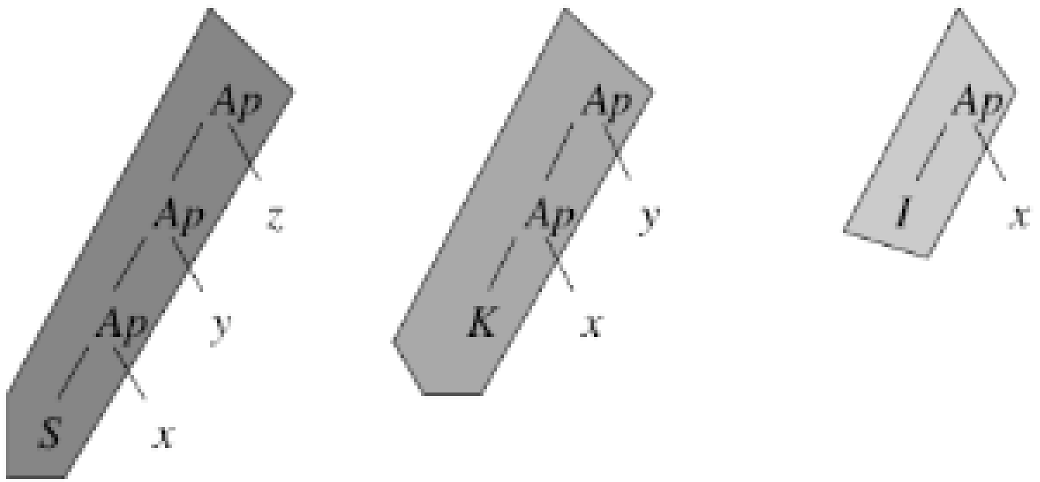
$$A(x, S(y)) \to S(A(x, y))$$

$$M(x, 0) \to 0$$

$$M(x, S(y)) \to A(M(x, y), x)$$



Combinatory Logic



$Ix \longrightarrow x$

$Kxy \longrightarrow x$

$Sxyz \longrightarrow xz(yz)$

orthogonal, hence confluent

SII(SII)
I(SII)(I(SII))
SII(I(SII))
I(I(SII))(I(I(SII)))
I(SII)(I(I(SII)))
SII(I(I(SII)))
I(I(I(SII)))(I(I(I(SII))))
I(I(SII))(I(I(I(SII))))
I(SII)(I(I(I(SII))))
SII(I(I(I(SII))))
I(I(I(I(SII))))(I(I(I(I(SII)))))
I(I(I(SII)))(I(I(I(I(SII)))))
I(I(SII))(I(I(I(I(SII)))))
I(SII)(I(I(I(I(SII)))))
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I(I(I(I(I(I(I(SII)))))))(I(I(I(I(I(I(I(SII))))))))

A=SSS
AAA
AAA
SSSAA
SA(SA)A
AA(SAA)
SSSA(SAA)
SA(SA)(SAA)
A(SAA)(SA(SAA))
SSS(SAA)(SA(SAA))
S(SAA)(S(SAA))(SA(SAA))
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SSS(A(SA(SAA)))(SAA(A(SA(SAA))))(S(SA(SAA))(A(SA(SAA))))(S(SAA)(SA(SAA)))
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A(SAA(A(SA(SAA))))(SAA(SAA(A(SA(SAA)))))(S(SA(SAA))(SAA(A(SA(SAA)))))(S(A(SA(SAA)))(SAA(A(SA(SAA)))))(S(SA(SAA))(A(SA(SAA))))(S(SAA)(SA(SAA)))

for Freek Wiedijks lambda
and CL calculator see

http://www.cs.vu.nl/~terese/lambda.html

REWRITE, REWRITE, REWRITE, REWRITE, REWRITE, ...*

Nachum DERSHOWITZ†

Department of Computer Science, University of Illinois at Urbana-Champaign, 1304 West Springfield Ave., Urbana, IL 61801–2987, U.S.A.

Stéphane KAPLAN‡

Department of Computer Science, Bar-Ilan University, Ramat-Gan 52100, Israel

Laboratoire de Recherche en Informatique, Université de Paris-Sud, Orsay 91405, France

David A. PLAISTED§

Department of Computer Science, University of North Carolina at Chapel Hill, Chapel Hill, NC 27514–3175, U.S.A.

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Abstract. We study properties of rewrite systems that are not necessarily terminating, but allow instead for transfinite derivations that have a limit. In particular, we give conditions for the existence of a limit and for its uniqueness and relate the operational and algebraic semantics of infinitary theories. We also consider sufficient completeness of hierarchical systems.

Is there no limit?
—Job 16:3

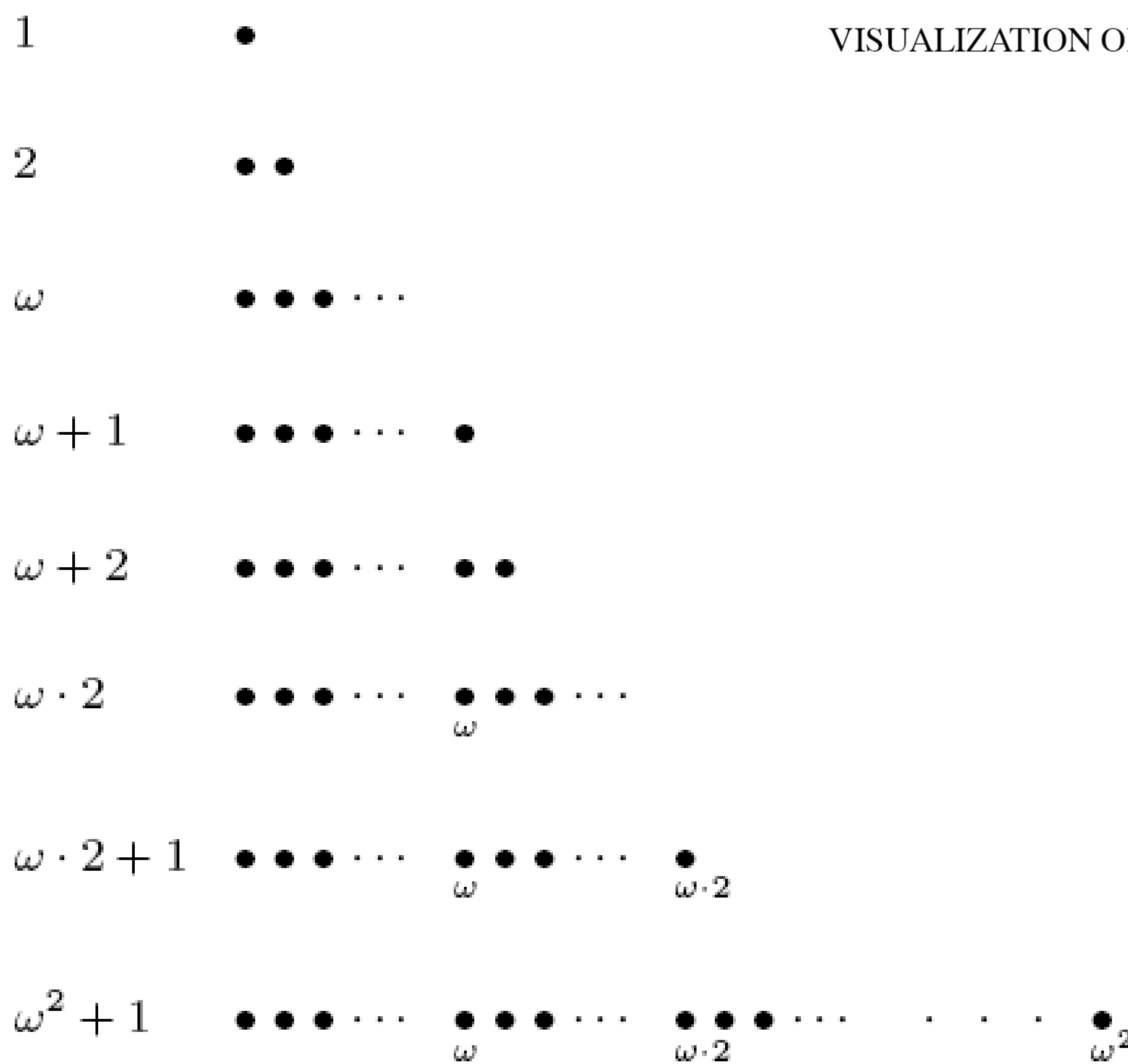
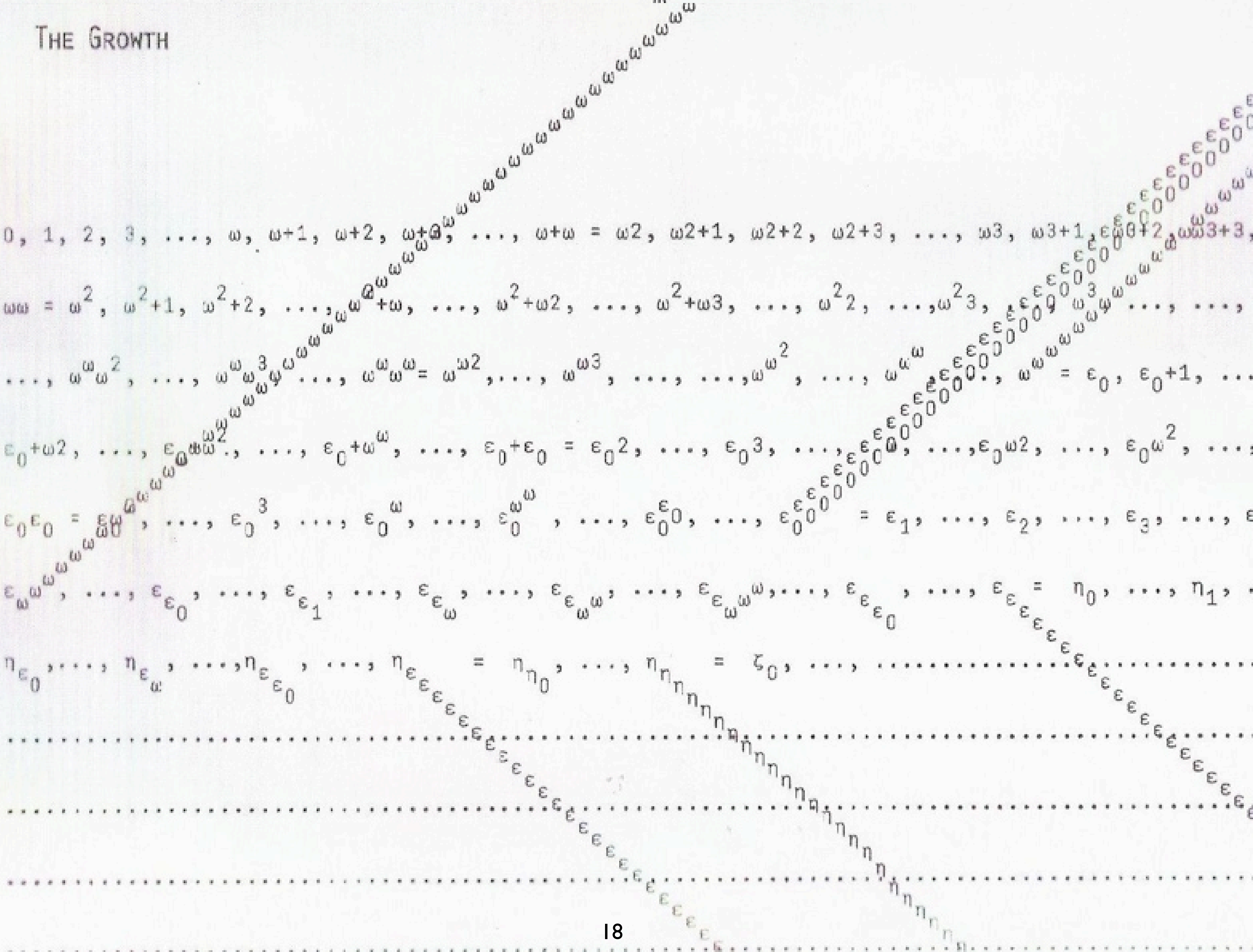
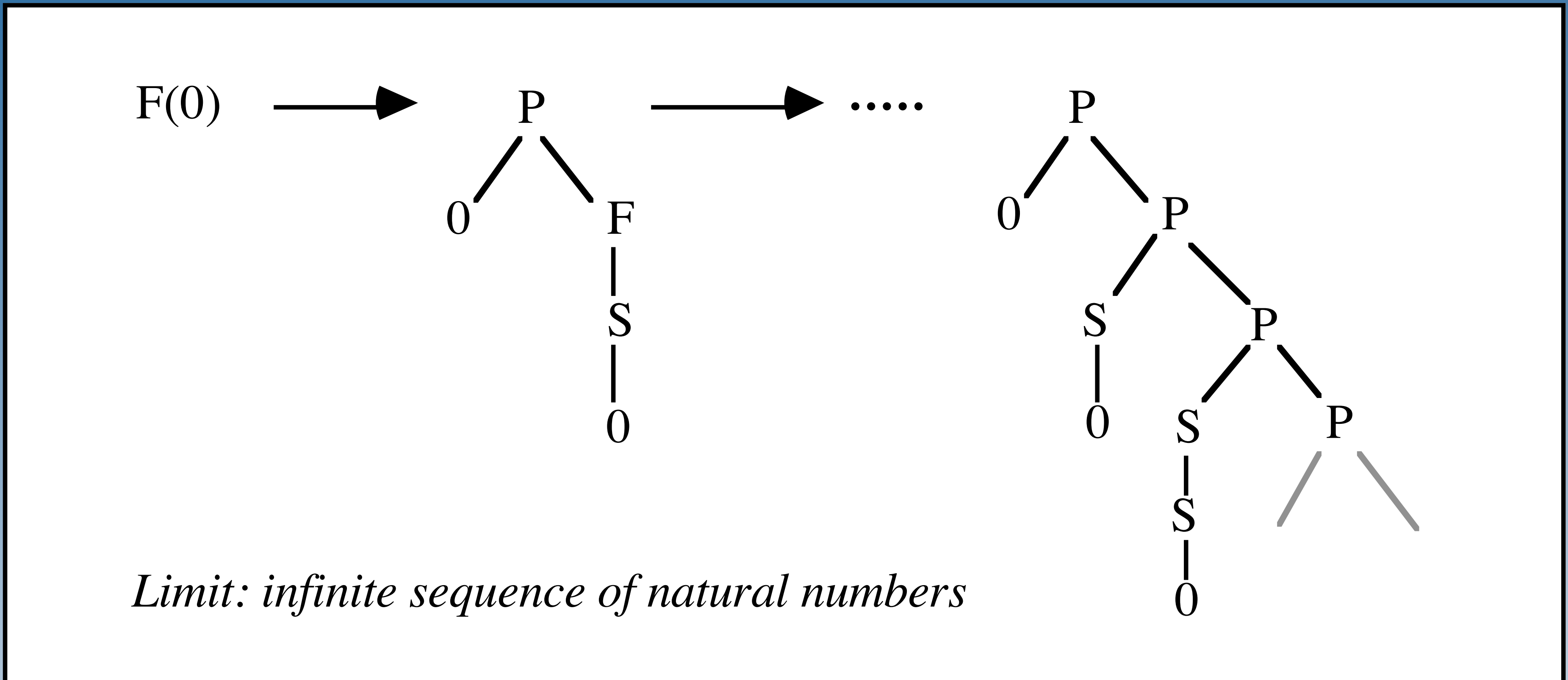
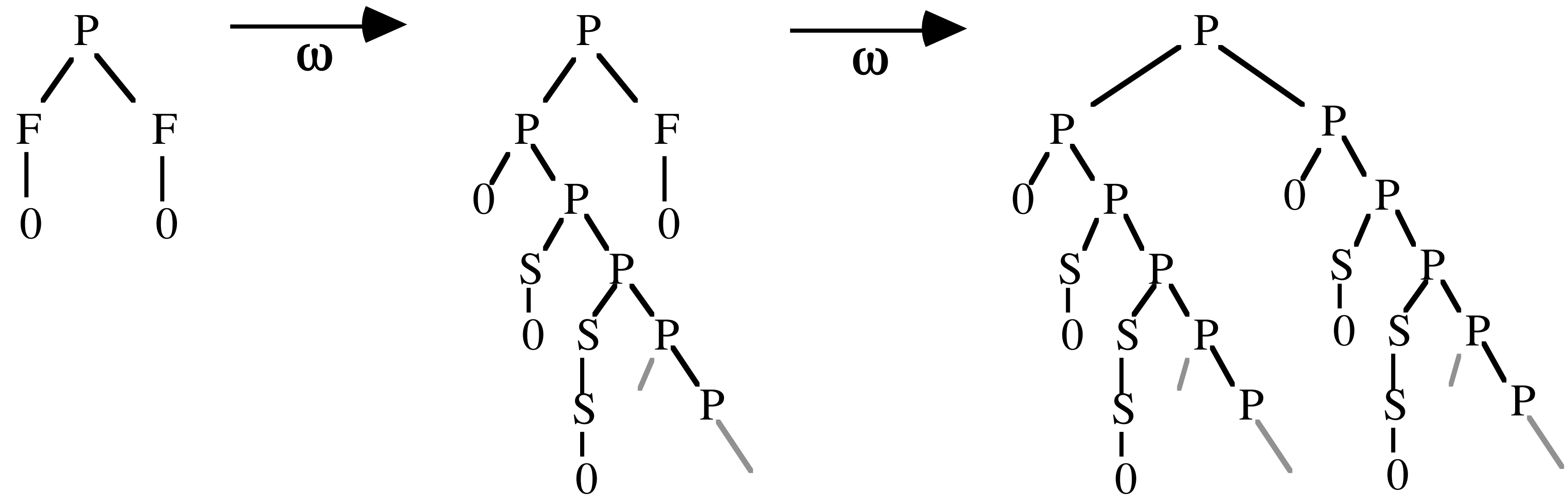


Fig. 3. Dot diagrams of the ordinals 1, 2, ω, $\omega+1$, $\omega+2$, $\omega\cdot 2$, $\omega\cdot 2+1$, and ω^2+1.


$0, 1, 2, 3, \ldots, \omega, \omega+1, \omega+2, \omega+3, \ldots, \omega+\omega = \omega 2, \omega 2+1, \omega 2+2, \omega 2+3, \ldots, \omega 3, \omega 3+1, \omega 3+2, \omega 3+3,$
$\omega\omega = \omega^2, \omega^2+1, \omega^2+2, \ldots, \omega^2+\omega, \ldots, \omega^2+\omega 2, \ldots, \omega^2+\omega 3, \ldots, \omega^2 2, \ldots, \omega^2 3, \ldots, \omega^3, \ldots, \ldots,$
$\ldots, \omega^{\omega^2}, \ldots, \omega^{\omega^3}, \ldots, \omega^{\omega^\omega} = \omega^{\omega 2}, \ldots, \omega^{\omega 3}, \ldots, \ldots, \omega^{\omega^2}, \ldots, \omega^{\omega^\omega}, \ldots, \omega^{\omega^{\cdot^{\cdot}}} = \varepsilon_0, \varepsilon_0+1, \ldots$
$\varepsilon_0+\omega 2, \ldots, \varepsilon_0+\omega^2, \ldots, \varepsilon_0+\omega^\omega, \ldots, \varepsilon_0+\varepsilon_0 = \varepsilon_0 2, \ldots, \varepsilon_0 3, \ldots, \varepsilon_0\omega, \ldots, \varepsilon_0\omega 2, \ldots, \varepsilon_0\omega^2, \ldots$
$\varepsilon_0\varepsilon_0 = \varepsilon_0^2, \ldots, \varepsilon_0^3, \ldots, \varepsilon_0^\omega, \ldots, \varepsilon_0^{\omega^\omega}, \ldots, \varepsilon_0^{\varepsilon_0}, \ldots, \varepsilon_0^{\varepsilon_0^{\cdot^{\cdot}}} = \varepsilon_1, \ldots, \varepsilon_2, \ldots, \varepsilon_3, \ldots,$
$\varepsilon_{\omega^\omega}, \ldots, \varepsilon_{\varepsilon_0}, \ldots, \varepsilon_{\varepsilon_1}, \ldots, \varepsilon_{\varepsilon_\omega}, \ldots, \varepsilon_{\varepsilon_{\omega^\omega}}, \ldots, \varepsilon_{\varepsilon_{\omega^{\omega^\omega}}}, \ldots, \varepsilon_{\varepsilon_{\varepsilon_0}}, \ldots, \varepsilon_{\varepsilon_{\varepsilon_{\cdot}}} = \eta_0, \ldots, \eta_1, \ldots$
$\eta_{\varepsilon_0}, \ldots, \eta_{\varepsilon_\omega}, \ldots, \eta_{\varepsilon_{\varepsilon_0}}, \ldots, \eta_{\varepsilon_{\varepsilon_{\cdot}}} = \eta_{\eta_0}, \ldots, \eta_{\eta_{\eta_{\cdot}}} = \zeta_0, \ldots$

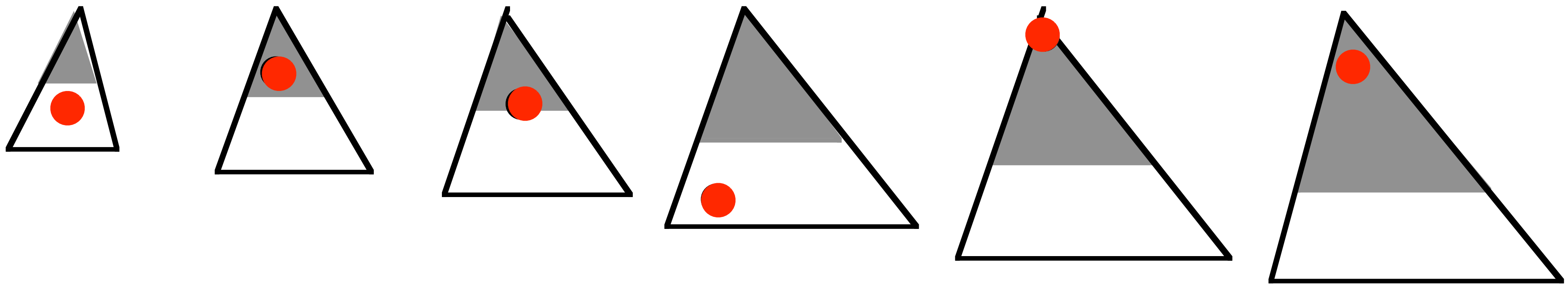


$$F(x) \rightarrow P(x, F(S(x)))$$

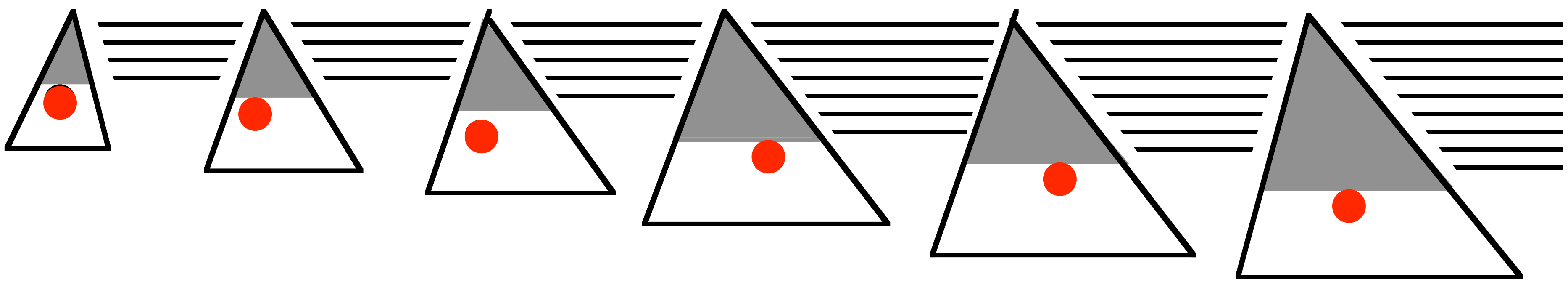


Transfinite reduction sequence of length $\omega + \omega$

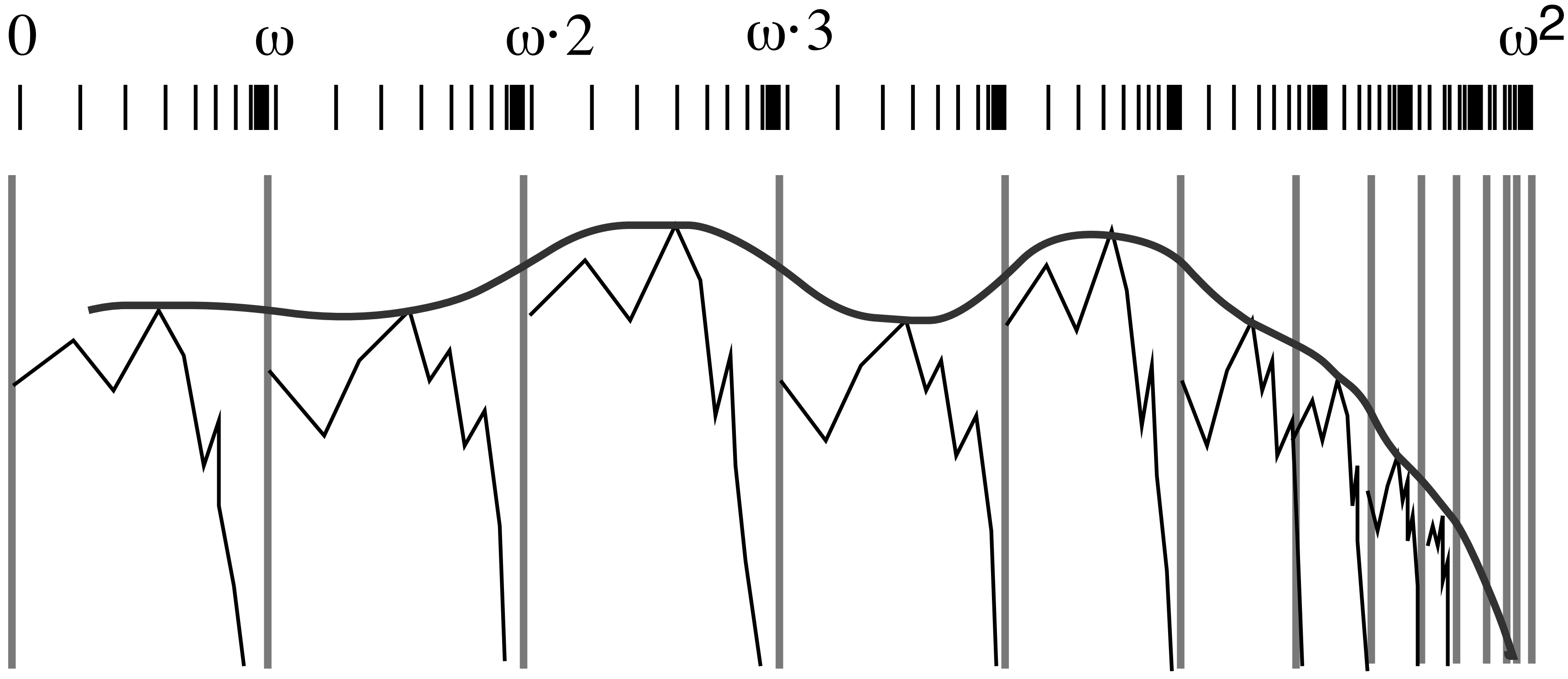
$$F(x) \to P(x, F(S(x)))$$



Cauchy converging reduction sequence: activity may occur everywhere

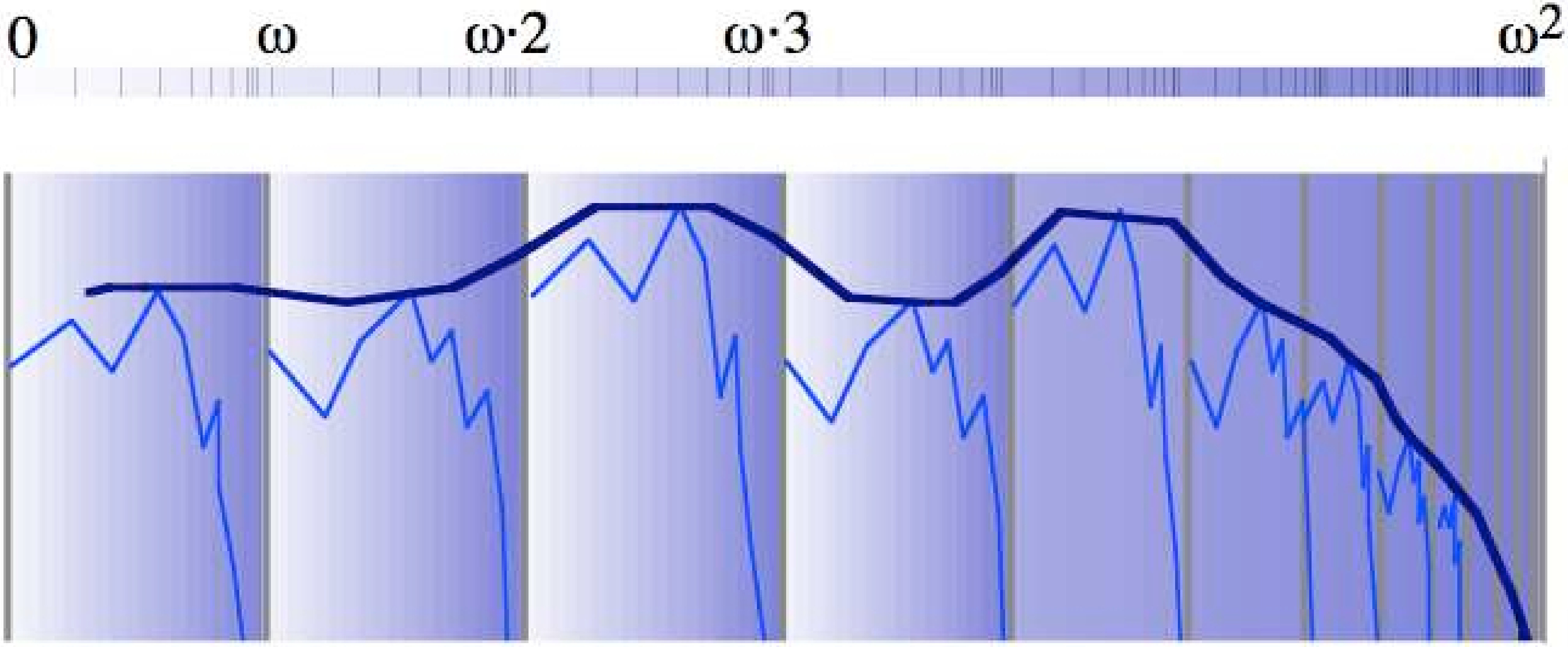


Strongly converging reduction sequence, with descendant relations



depth of contracted redex tends to infinity at each limit ordinal

But we may avoid transfinite ordinals by COMPRESSION.



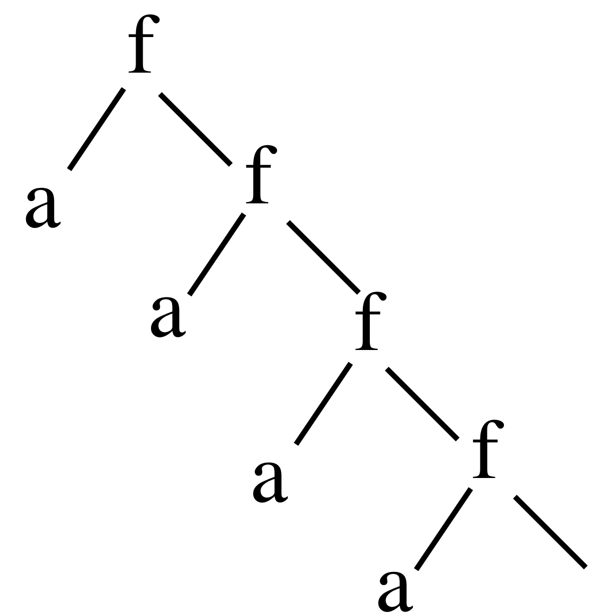
A transfinite reduction
Depth of redex activity goes down at each limit ordinal

But if you don't like ordinals, there is for orthogonal TRSs the Compression Lemma:

every reduction of length α can be compressed to ω or less.

use dove-tailing

Every countable ordinal can be the length of an infinite reduction.
Consider the TRS
$\{c \longrightarrow f(a, c)$ and $a \longrightarrow b\}$



finitary rewriting	infinitary rewriting
finite reduction	*strongly convergent reduction*
infinite reduction	*divergent reduction*
normal form	*(poss. infinite) normal form*
CR: finite coinitial reductions can be joined	*CR^∞: infinite coinitial reductions can be joined*
UN: coinitial reductions to nf end in same nf	*UN^∞: coinitial reductions to nf end in same nf*
SN: there are no infinite reductions	*SN^∞: there are no divergent reductions*
WN: there is a reduction to nf	*WN^∞: there is a reduction to nf*

$A(x, 0) \to x$

$A(x, S(y)) \to S(A(x, y))$

$M(x, 0) \to 0$

$M(x, S(y)) \to A(M(x, y), x)$

$\infty \to S(\infty)$

coinductive natural numbers

data

0, S(0), S(S0)),...

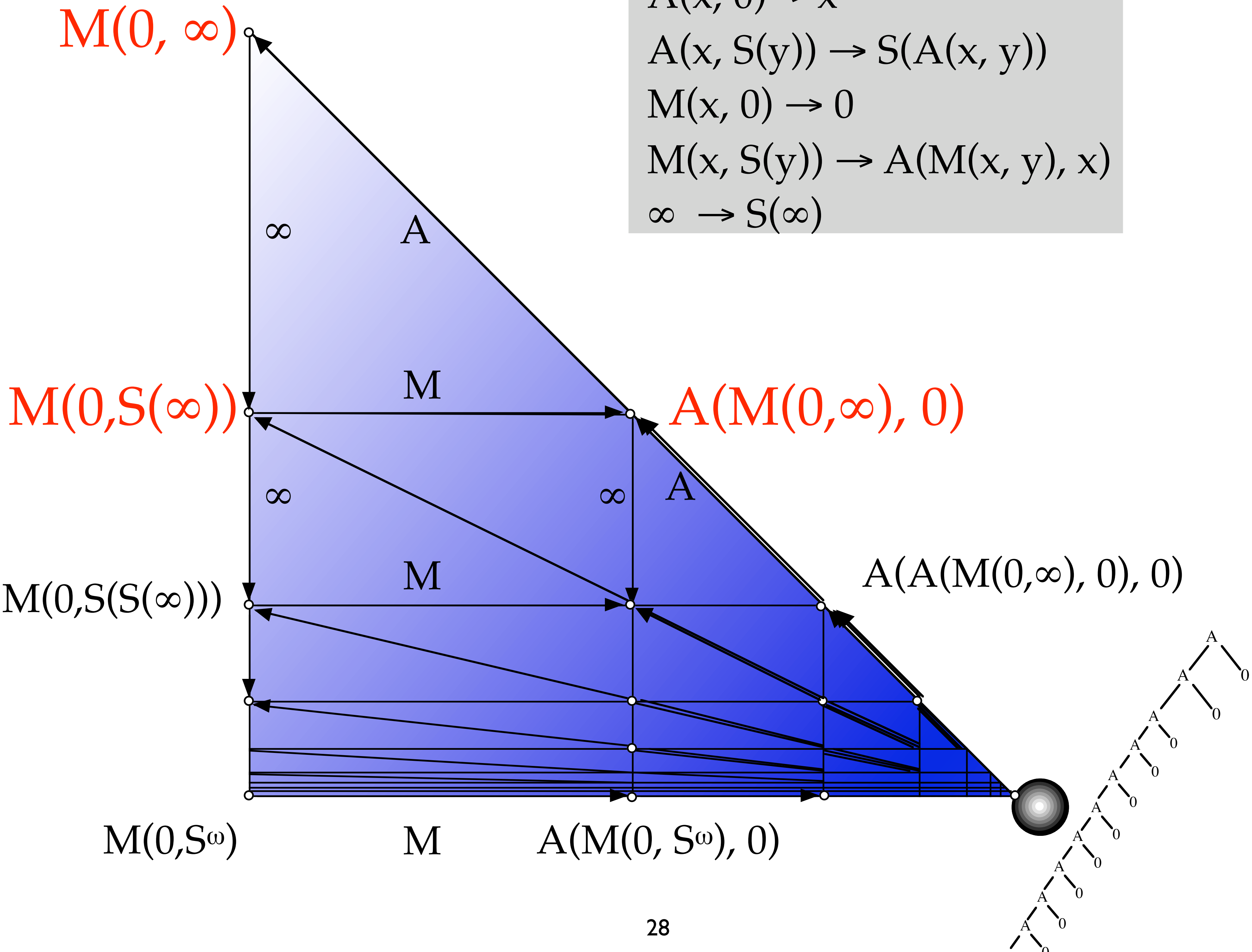
codatum

S(S(S(S...

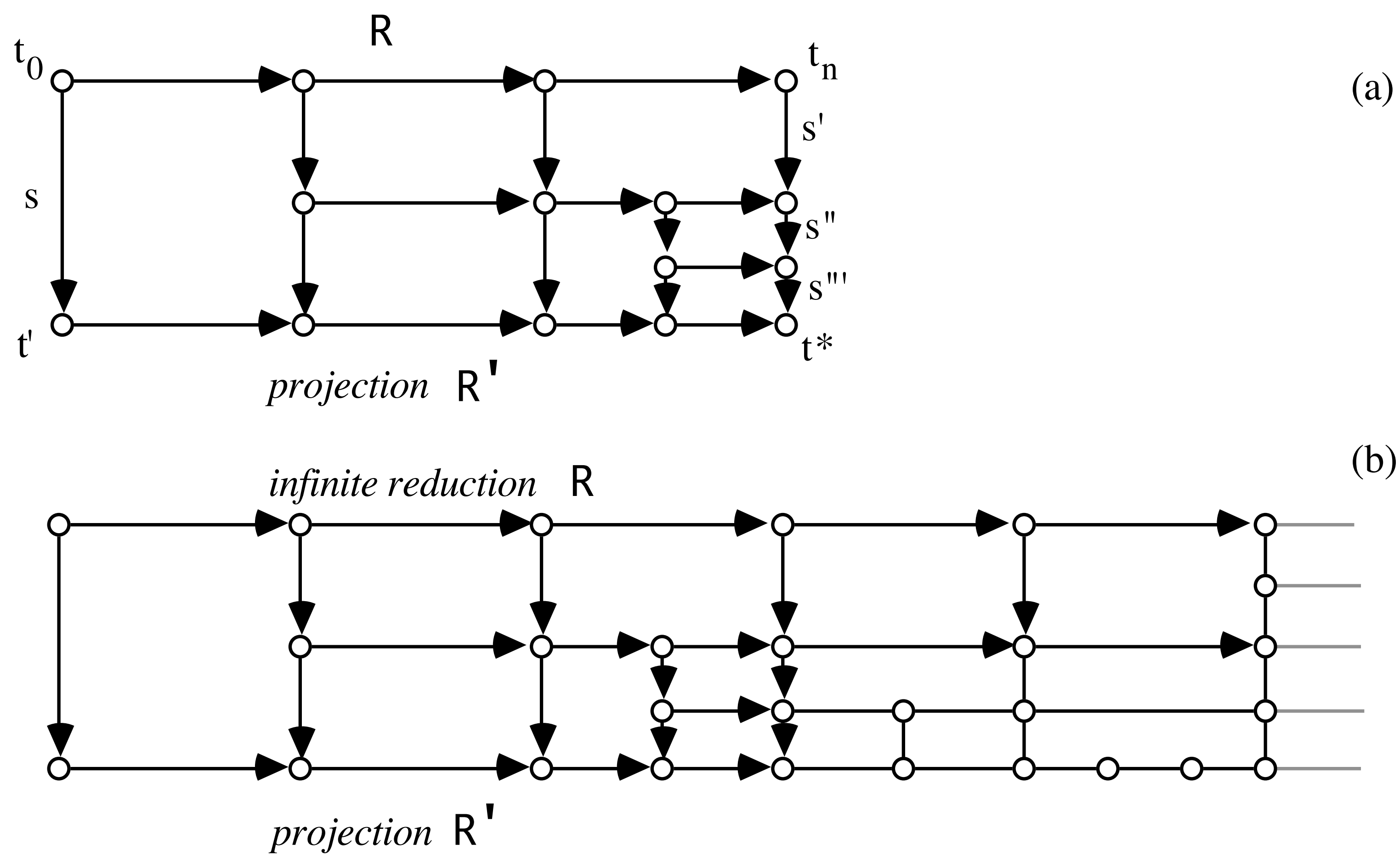
orthogonal constructor TRS
with one collapsing rule

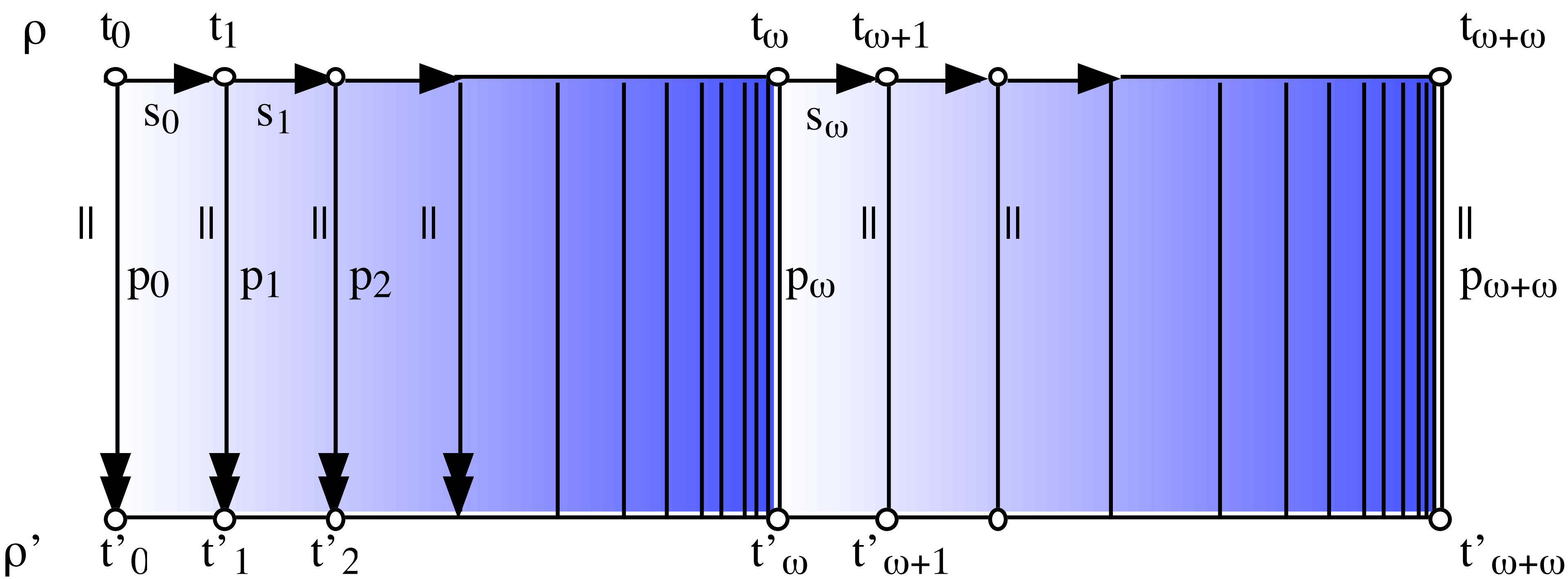
$$\begin{tikzcd}
 & SN^{\infty} \arrow[d, shift right] & CR^{\infty} \arrow[d] \\
AC & WN^{\infty} \arrow[l] \arrow[u, shift right, "NE"'] & UN^{\infty}
\end{tikzcd}$$

$$A(x, 0) \to x$$
$$A(x, S(y)) \to S(A(x, y))$$
$$M(x, 0) \to 0$$
$$M(x, S(y)) \to A(M(x, y), x)$$
$$\infty \to S(\infty)$$



Parallel Moves Lemma

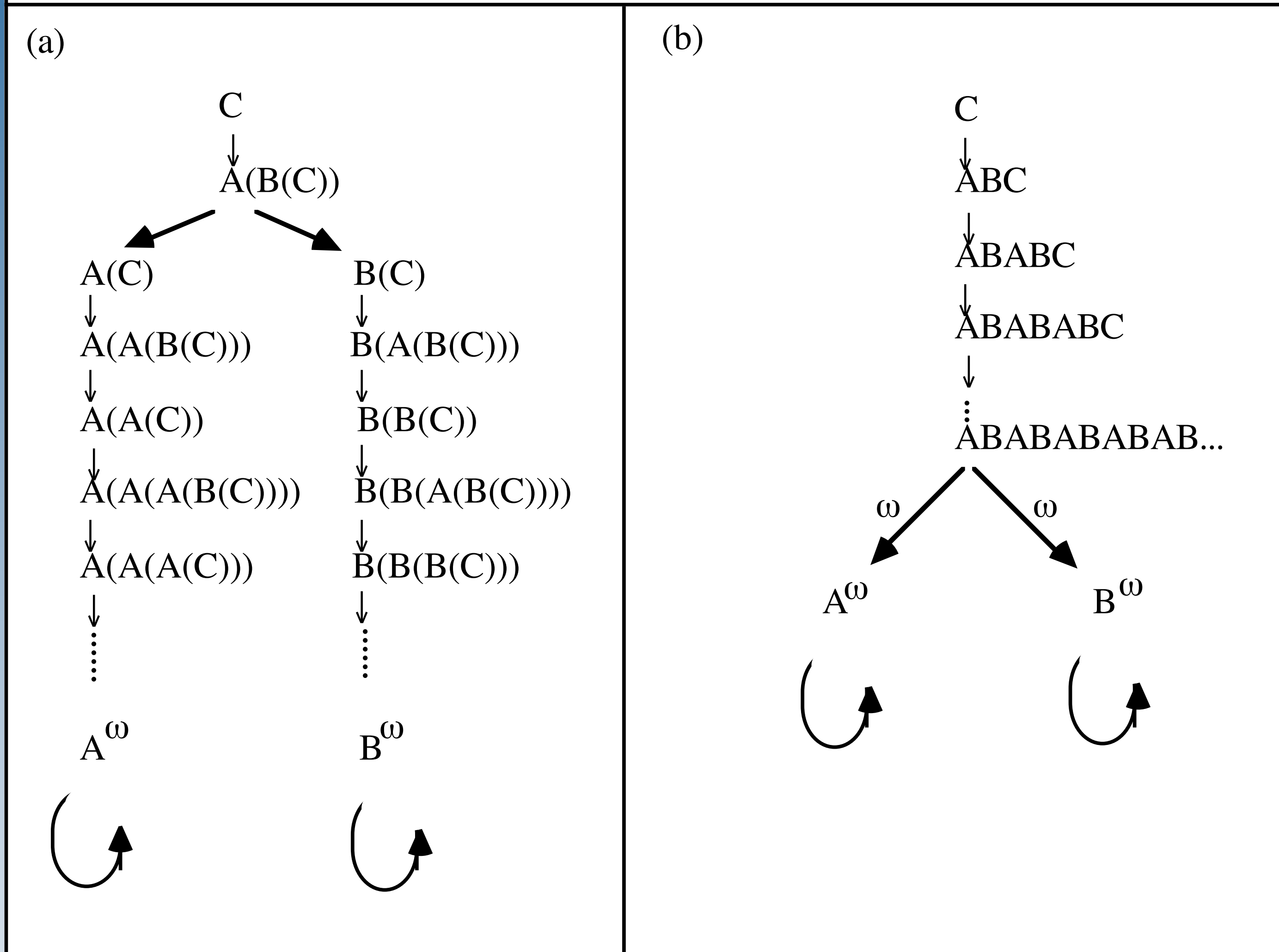




PML$^\infty$ *For first order infinitary term rewriting we have the infinitary Parallel Moves Lemma PML$^\infty$*

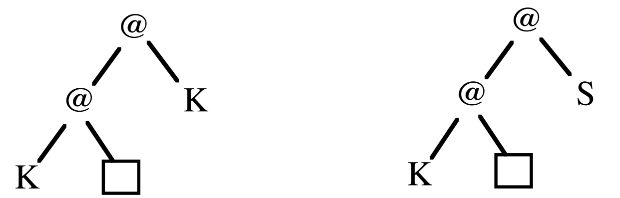
not CR^∞

$$
\begin{aligned}
A(x) &\to x \\
B(x) &\to x \\
C &\to A(B(C))
\end{aligned}
$$

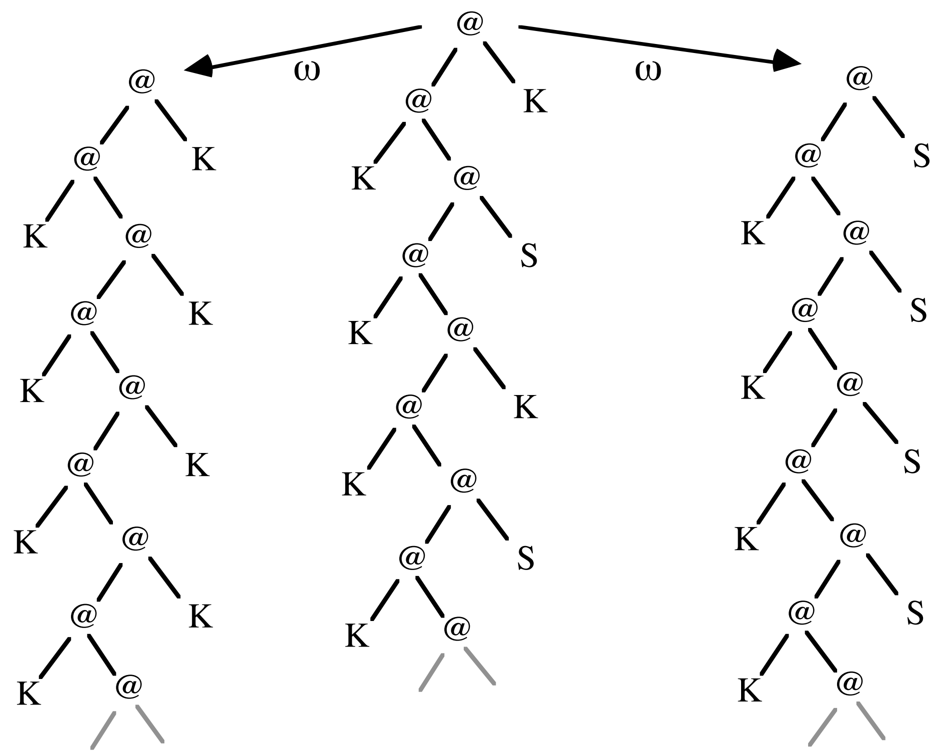


Failure of infinitary confluence

Sxyz	$\rightarrow$ xz(yz)	@(@(@(S, x), y), z)	$\rightarrow$ @(@(x, z), @(y, z))
Kxy	$\rightarrow$ x	@(@(K, x), y)	$\rightarrow$ x

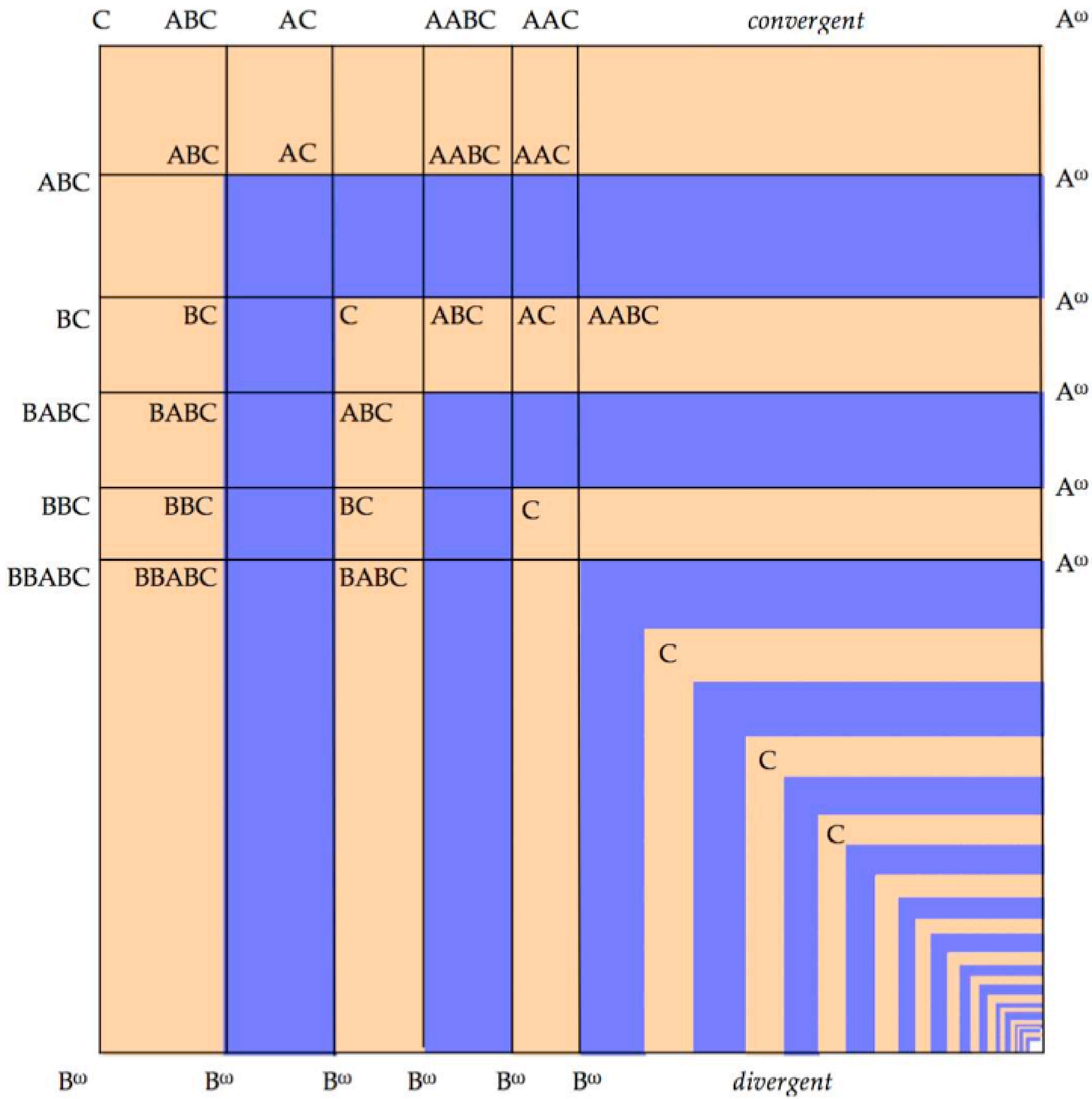


collapsing contexts



Failure of infinitary confluence for Combinatory Logic

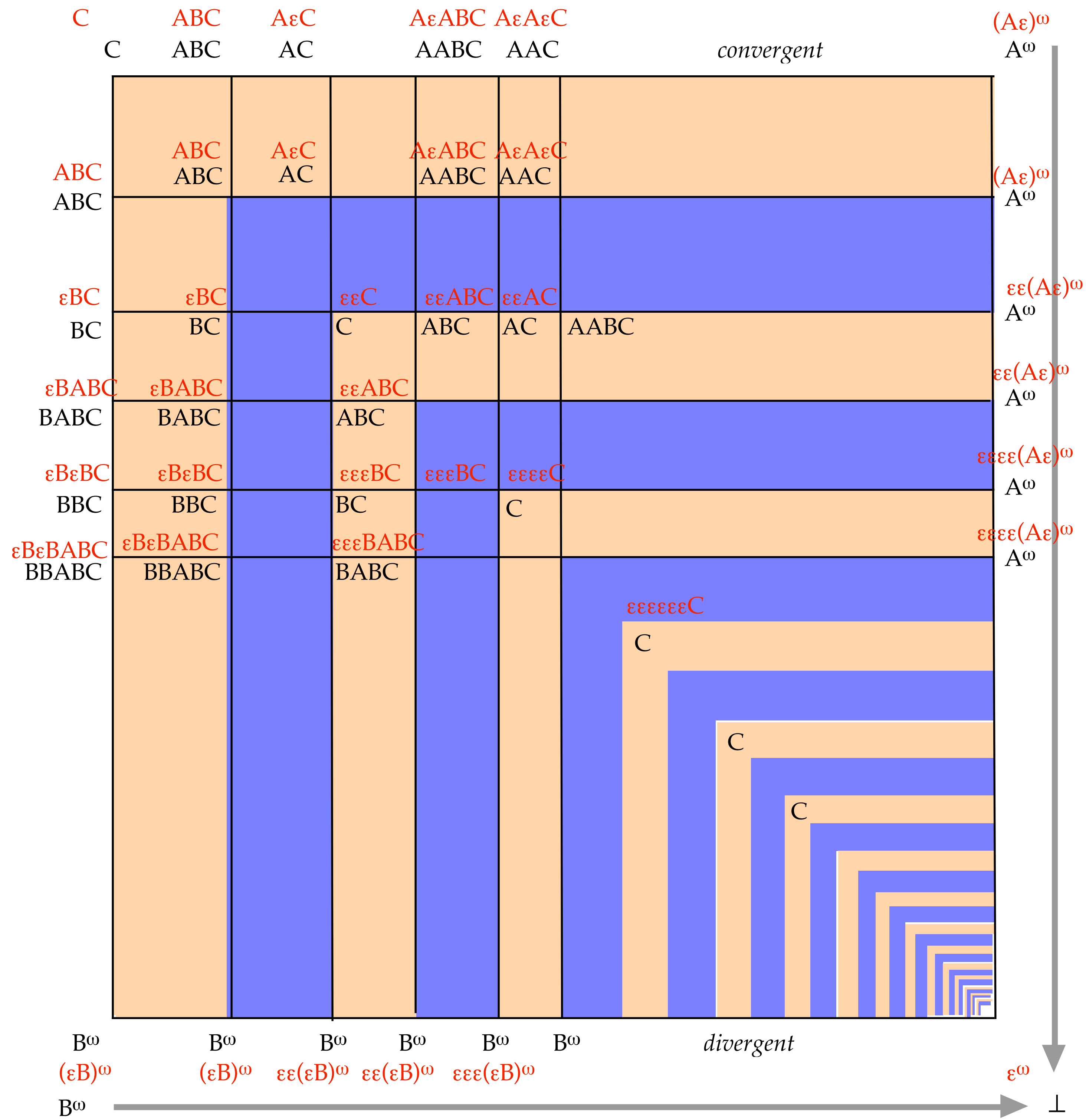
Failure of CR^{∞}



$A(x) \rightarrow x$

$B(x) \rightarrow x$

$C \rightarrow A(B(C))$


C
C
ABC
ABC
AεC
AC
AεABC
AABC
AεAεC
AAC
convergent
(Aε)ω
Aω
ABC
ABC
AεC
AC
AεABC
AABC
AεAεC
AAC
ABC
ABC
(Aε)ω
Aω
εBC
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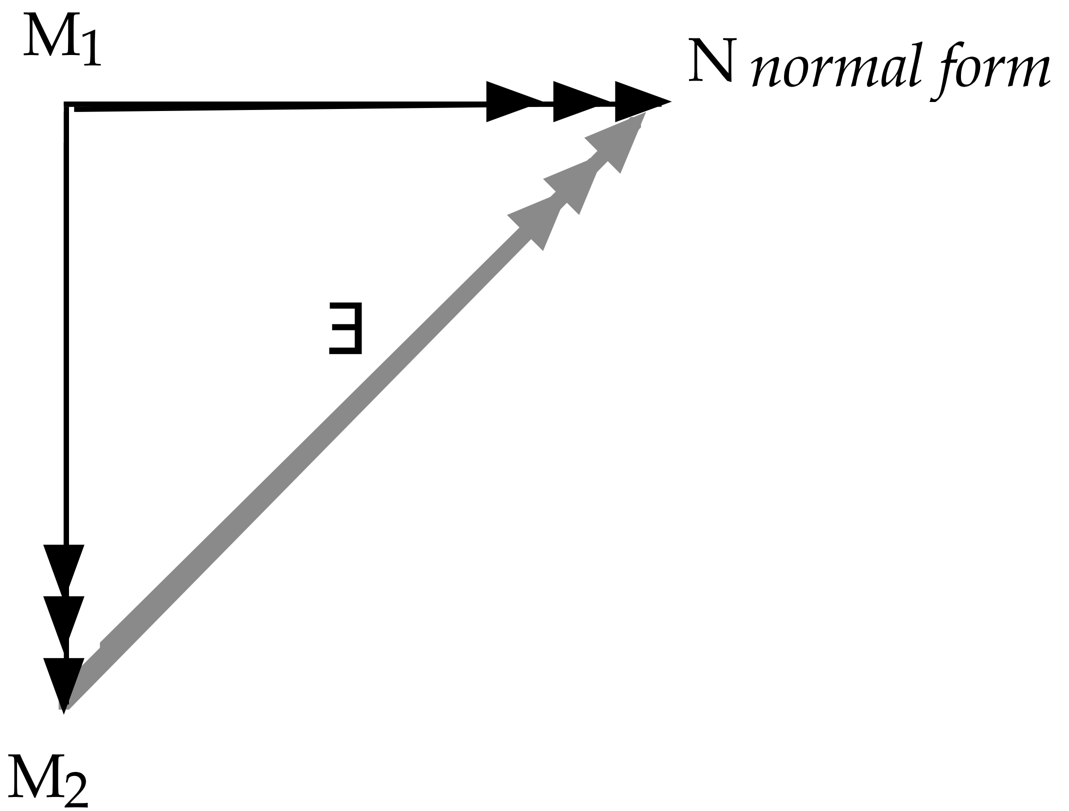
Bω
εεε(εB)ω
Bω
divergent
εω
Bω
⊥

for OTRSs: UN^{∞}.

Corollary: Dershowitz et al:
for OTRSs $SN^{\infty} \Rightarrow CR^{\infty}$.

Proof: as for finite case
SN & UN => CR



M1
N *normal form*
∃
M2

Confluence in infinitary rewriting

	PML	CR	UN	PML$^\infty$	CR$^\infty$	UN$^\infty$
OTRS	*yes*	*yes*	*yes*	*yes*	*no*	*yes*
w.o. TRS	*yes*	*yes*	*yes*	?	*no*	?
$\lambda\beta$	*yes*	*yes*	*yes*	*no*	*no*	*yes*
OCRS	*yes*	*yes*	*yes*	*no*	*no*	?

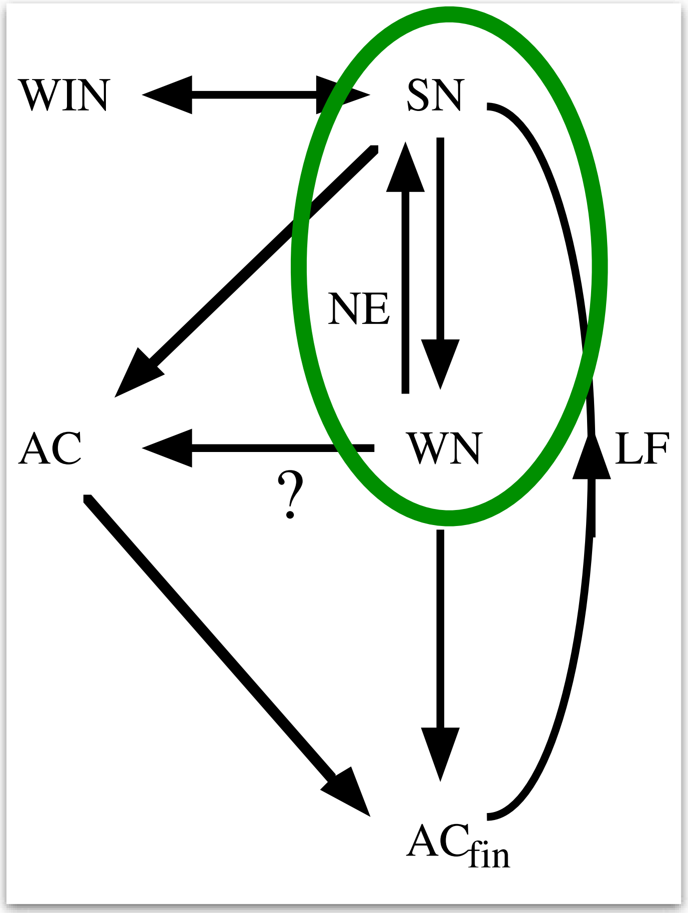
by CR$^\infty$ for a quotient of $\lambda\beta^\infty$, e.g. mute terms, or hypercollapsing terms, and applying an abstract lemma of de Vrijer.

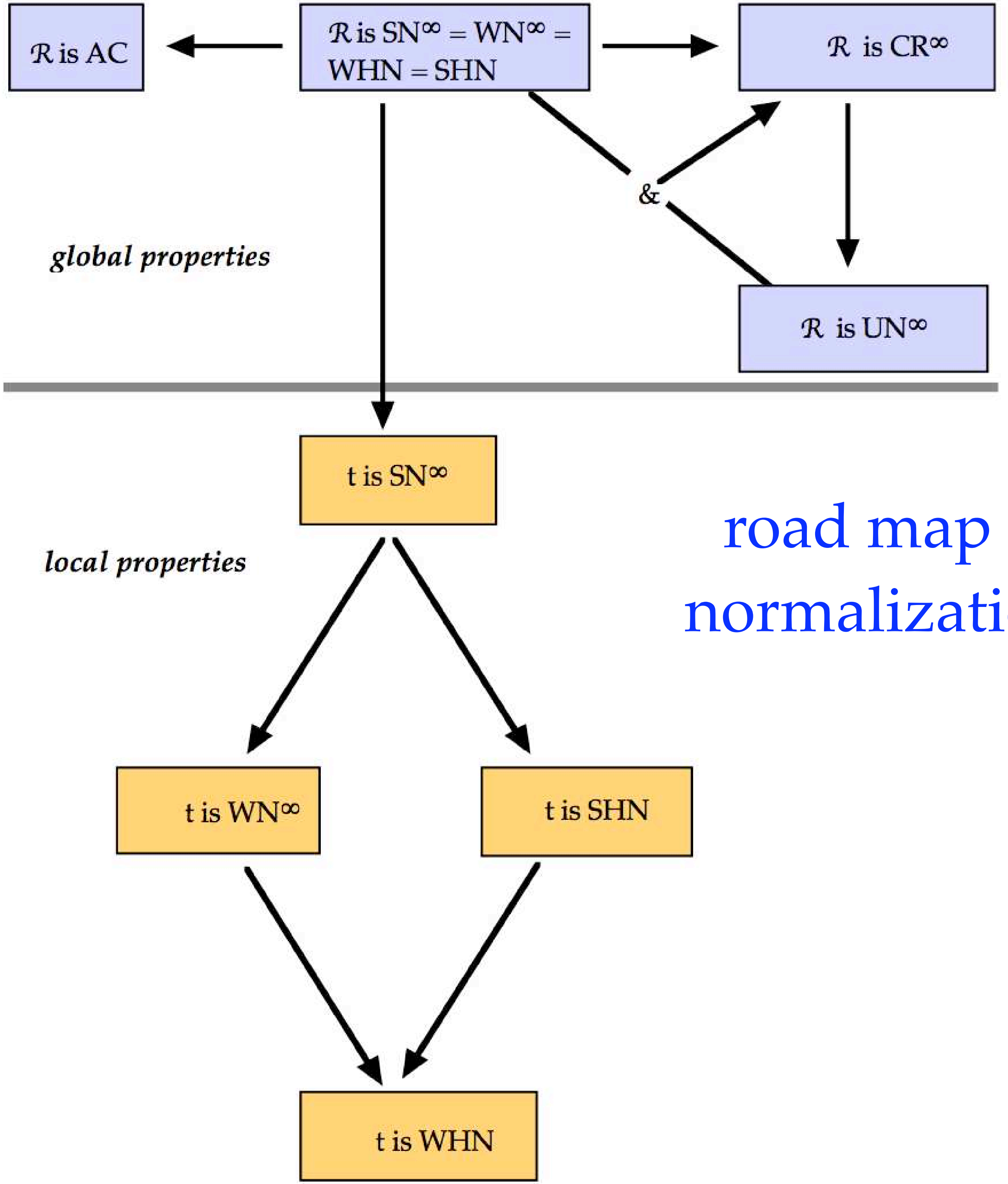
Let (A, $\rightarrow_1$) and (B, $\rightarrow_2$) be two ARSs with A included in B, reduction $\rightarrow_1$ included in $\rightarrow_2$, normal forms nf(A) included in nf(B). Then CR for B implies UN for A.

	PML	CR	UN	PML$^\infty$	CR$^\infty$	UN$^\infty$
orthogonal TRS	*yes*	*yes*	*yes*	*yes*	*no*	*yes*
weakly orthogonal TRS	*yes*	*yes*	*yes*	*yes (in prep.)*	*no*	*yes (in prep.)*
λβ-calculus	*yes*	*yes*	*yes*	*no*	*no*	*yes*
orthogonal CRS	*yes*	*yes*	*yes*	*no*	*no*	*yes*

Overview

- AC: no cycles
- WIN: weak innermost normalisation
- SN: strong normalisation
- NE: non-erasing
- AC_{fin}: no cycles in finite graphs
- LF: locally finite: all graphs are finite





road map of infinitary normalization properties

How to define SN^{∞} and WN^{∞}?

WN^{∞} is easy: There is a possibly infinite reduction to the possibly infinite normal form.

SN^{∞} : all reductions will eventually terminate in the normal form. The only way such a reduction could fail to reach a normal form, is that it stagnates at some point in the tree which is developing, for infinitely many steps. Then no limit can be taken.

Good and bad reductions. In ordinary rewriting the finite reductions are good, they have an end point, and the infinite ones are bad, they have no end point.

Same in infinitary rewriting. The good reductions are the ones that are strongly convergent, they have an end point. E.g. $a \to b(a)$ reaches after ω steps the end point b^{ω}.

The bad reductions (divergent, stagnating) are the ones without an end point. Their reductions may be long, a limit ordinal long, but there they fail.

SN^{∞} states that there are no bad reductions.

In other words: say we select at random in each step a redex and perform this step. We can go on until we reach a limit ordinal. At that point we look back, and if the reduction was strongly convergent we take the limit and go on. If not, we stop there and we had a bad reduction.

CLAIM: we can then identify a stagnating term, a term where infinitely often a root step was performed.

THEOREM. $SN^{\infty} \Leftrightarrow WN^{\infty}$

PROOF.
$\Rightarrow$ is clear.

$\Leftarrow$: suppose WN^{∞} and not SN^{∞}.
So there is a stagnating reduction.
So there is a term with an infinite reduction in which infinitely many root steps occur. Such a term does not have an (infinite) normal form,
i.e. not WN^{∞}. *This uses the Head Normalization Theorem generalized to infinite terms*

Head Normalization Theorem

A *head step* is a reduction step that takes place at the head or the root. This means that the redex contracted coincides with the whole term t. Notation: $t \to_h t'$.

A step which is not a head step is called an *internal* step; notation $t \to_i t'$.

Term t is a head normal form (hnf) when it does not reduce to a redex; has a hnf when it reduces to one.

THEOREM. *Let* R *be an orthogonal TRS and let* t *be a term in* R *with a reduction* $t \to t' \to t'' \to \ldots$ *containing infinitely many root steps.*

(i) *Then* t *has no head normal form.*

(ii) *A fortiori, t has no normal form.*

THEOREM. $SN^{\infty} \Rightarrow AC$

Waldmann: S-terms $\models SN^{\infty}$

COROLLARY: *S-terms are acyclic*

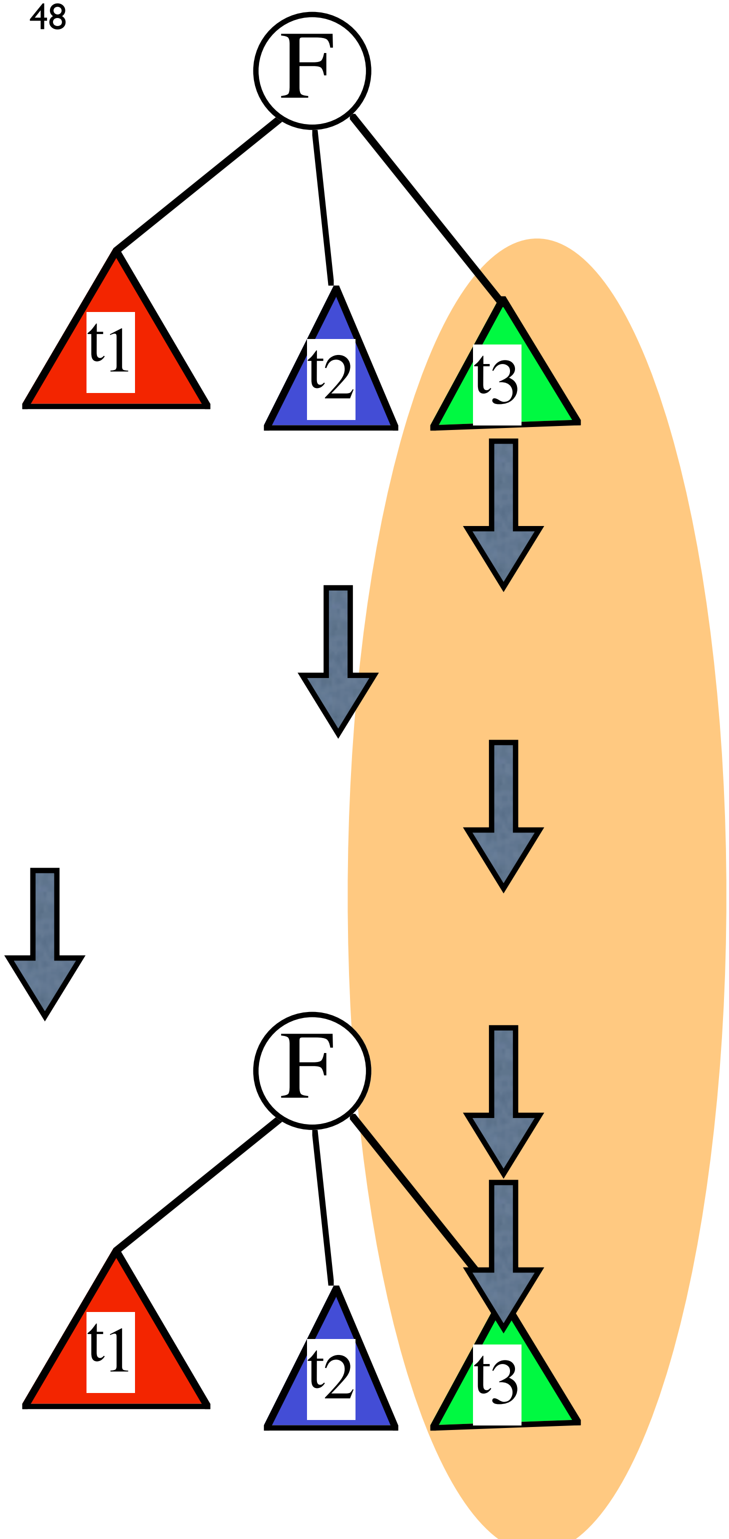
THEOREM. *For orthogonal TRSs: WN ⇒ AC*

PROOF. Will prove ¬AC ⇒ ¬WN. Suppose ¬AC. So there is a cycle. Now take a minimal cyclic term t, minimal with respect to the length. So there is a cyclic reduction $\mathcal{C}$: t → ...→ t, and all terms shorter than t are not cyclic.

Claim. One of the steps of *C* is a root step.

Proof of the claim. Let t have the form $F(t_1,\ldots,t_n)$ for some n. Suppose the claim is not true. So the root symbol F is 'frozen', not active, and all steps in *C* take place in the subterms $t_1,\ldots,t_n$. There must be a step done in *C*, say in t_i. Now we take out of *C* all the steps in t_i. They are not influenced by the the other steps in *C*. But then these steps in *C* constitute a cycle $t_i \to \ldots \to t_i$, contradicting the minimality of t.

Now we unwind the cycle into an infinite reduction *C*; *C*; *C*; ... *C*; ..., that is *C* repeated infinitely often, notation C^ω. By the Claim the reduction C^ω has infinitely many root steps. Hence, by the Head Normalisation Theorem, the starting term t does not have head normal form, and a fortiori no normal form. That is, we have proved ¬WN.



top remains frozen

hence smaller cycle

Related observation by Vincent van Oostrom:

A term allowing a trivial head step is not normalizing in a weakly orthogonal TRS.

Here a step is ‘trivial’ if it is of the form $t \to t$, so a one-step cycle.

Dershowitz, Kaplan, Plaisted 91

THEOREM. $SN^{\infty} \Rightarrow CR^{\infty}$

Failure of Newman's Lemma, infinitary version

This example establishes that we do *not* have the implication
WCR & $SN^{\infty} \Rightarrow CR^{\infty}$. In other words, the infinitary version of

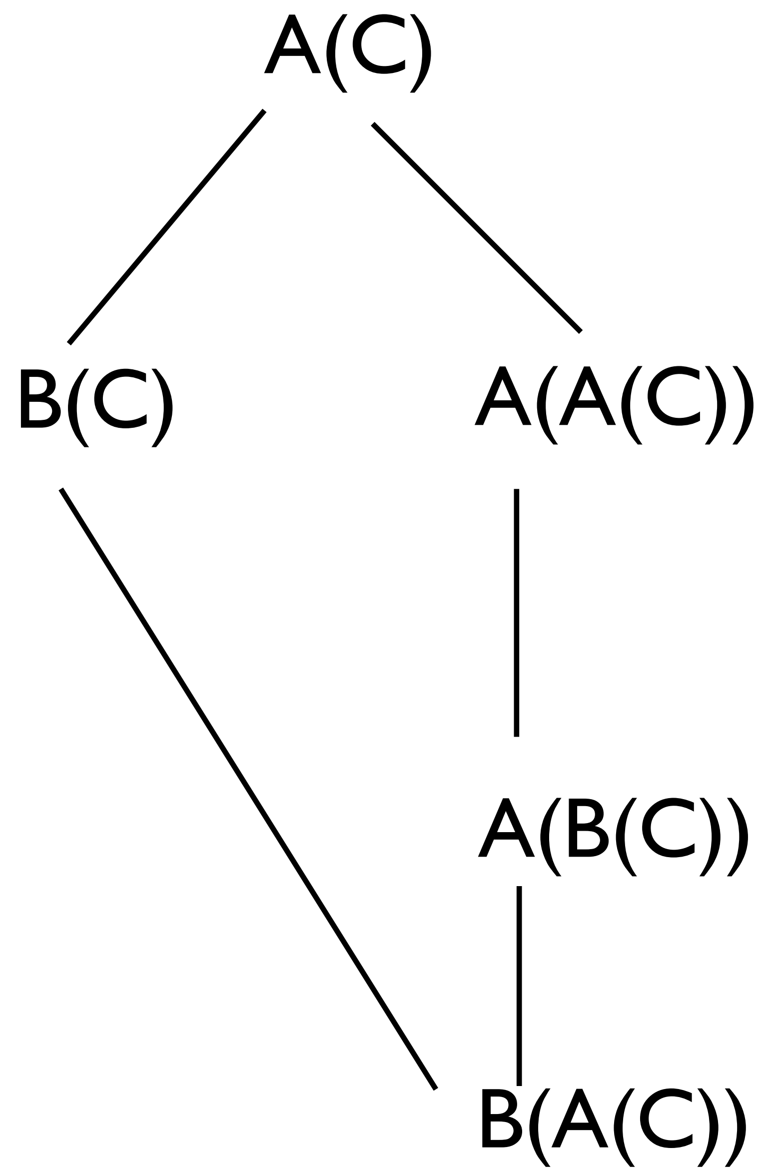
Newman's Lemma WCR & SN $\Rightarrow$ CR
for abstract reduction systems (ARSs) fails for infinitary term rewriting.

Consider the TRS with rules

$$C \to A(C)$$
$$A(C) \to B(C)$$
$$A(B(x)) \to B(A(x)).$$

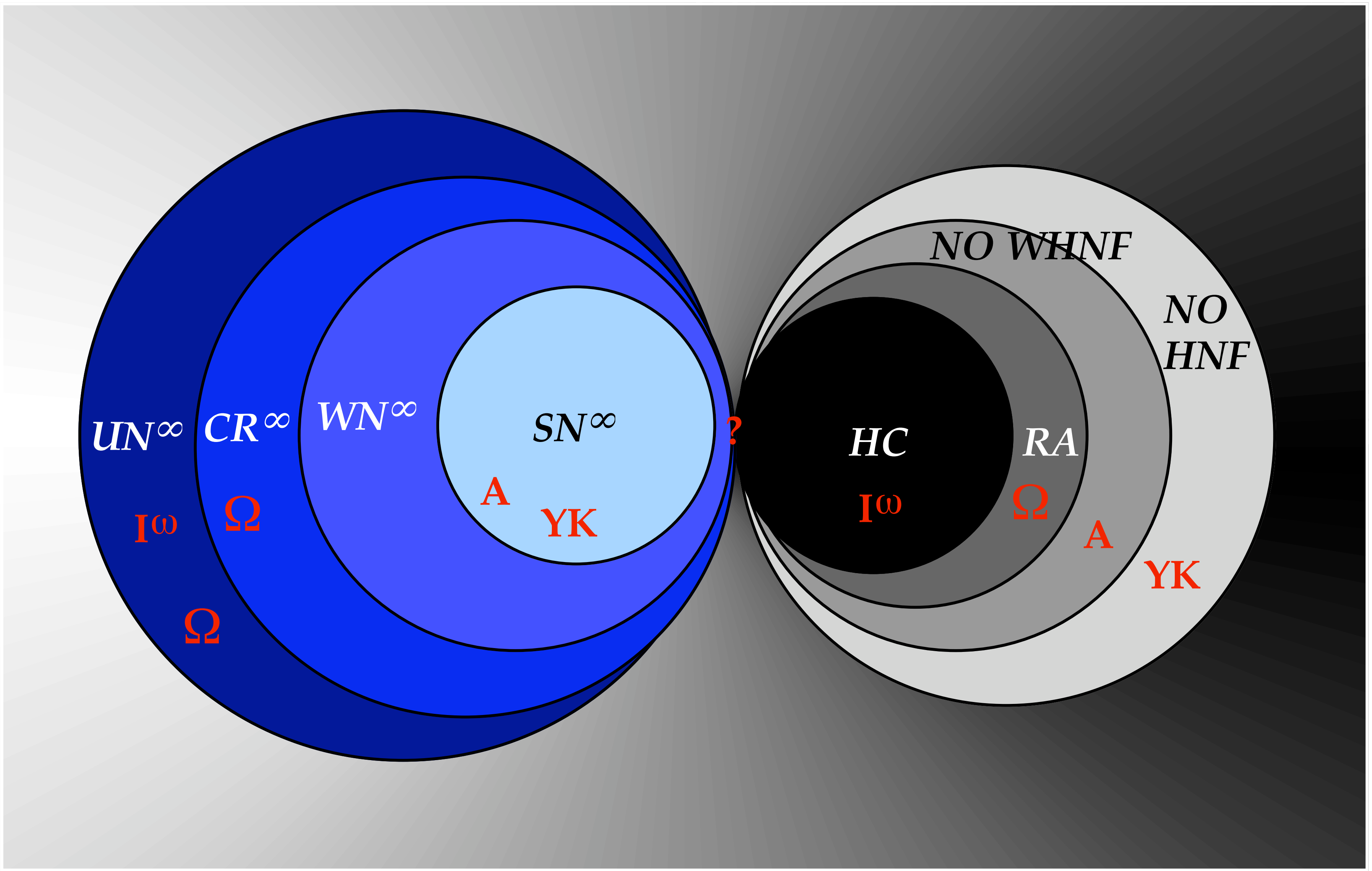
Note that we do not have $A(x) \to B(x)$! Now it is easily checked that WCR holds, by looking at the critical pairs and applying
Huet's Lemma stating that if all critical pairs are convergent, we have WCR.
We also have SN^{∞}; proving that is a nice exercise. But CR^{∞} fails, as C reduces in ω steps to
A^{ω} and B^{ω}, both infinite normal forms.

$C \to A(C)$
$A(C) \to B(C)$
$A(B(x)) \to B(A(x)).$



Huet's critical pair lemma

WCR

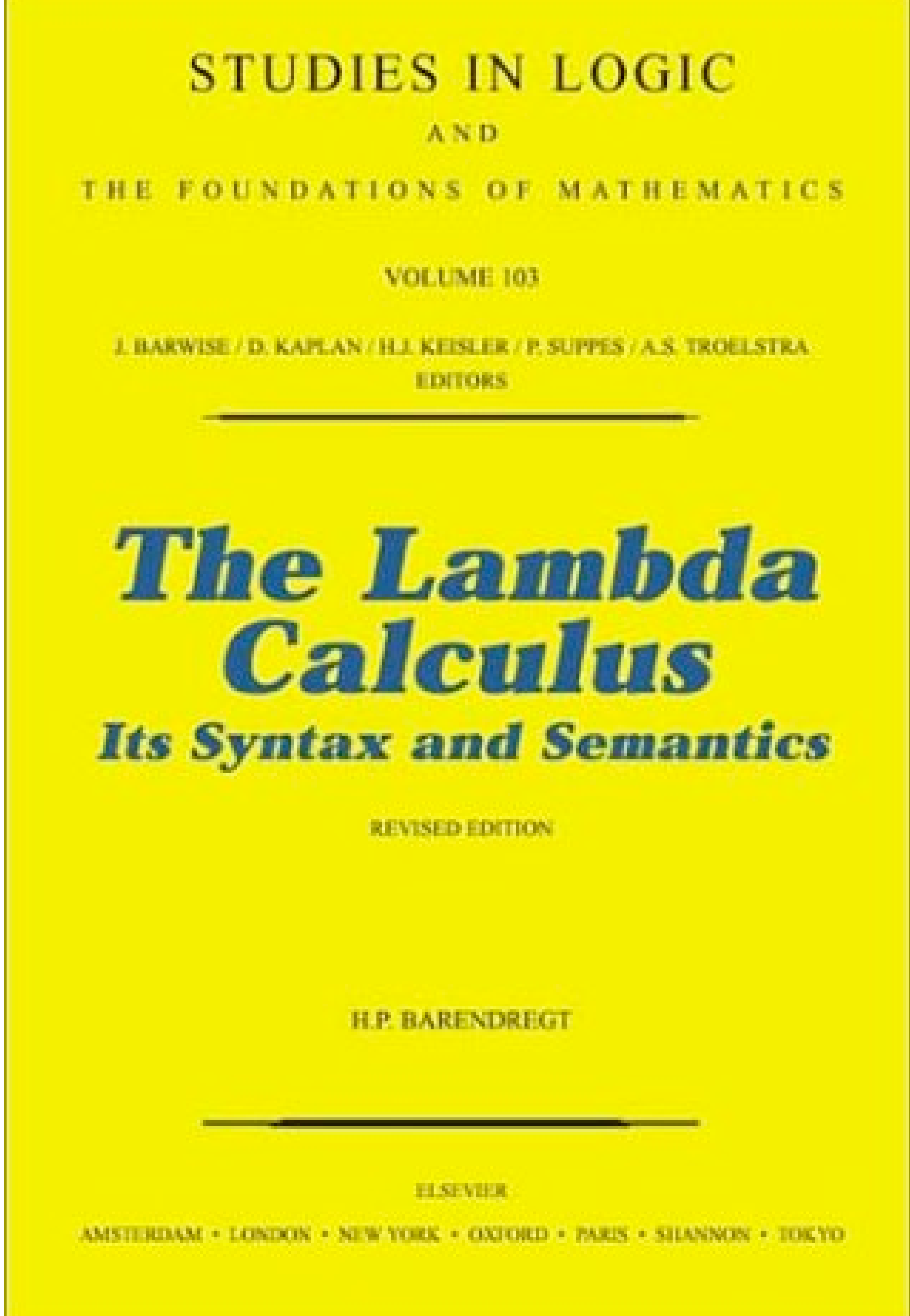


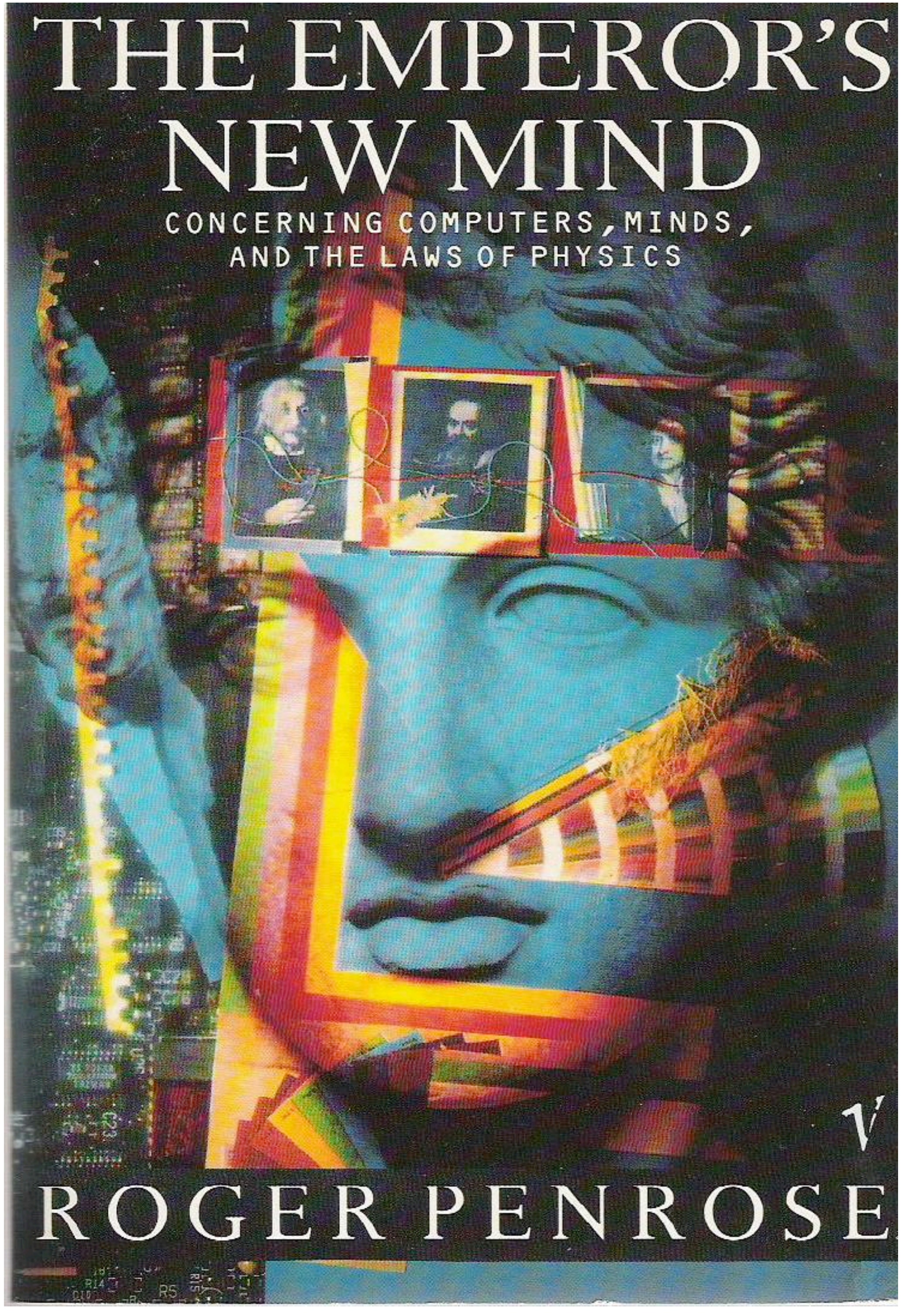
UN^{∞}
CR^{∞}
WN^{∞}
SN^{∞}
I^{ω}
Ω
Ω
A
YK
?
NO WHNF
NO HNF
HC
RA
I^{ω}
Ω
A
YK

Lambda Calculus

$$\boxed{(\lambda x.Z(x))Y \longrightarrow Z(Y)}$$

Turing complete




THE EMPEROR'S
NEW MIND
CONCERNING COMPUTERS, MINDS,
AND THE LAWS OF PHYSICS
ROGER PENROSE

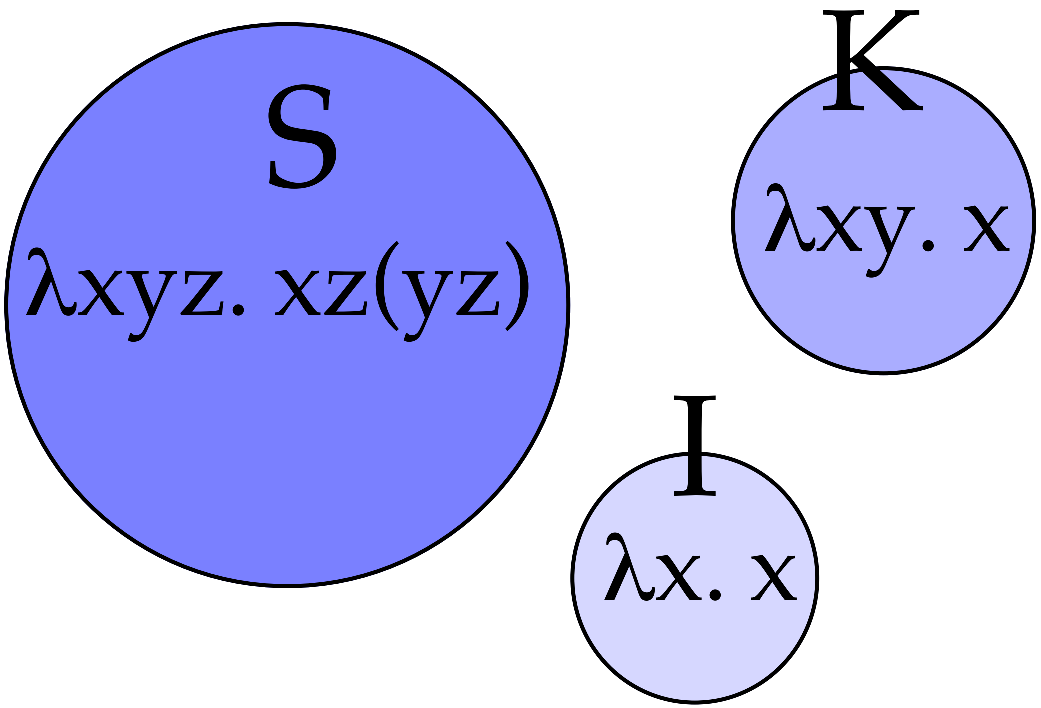
Combinatory Logic

$$\mathrm{I}x \longrightarrow x$$

$$\mathrm{K}xy \longrightarrow x$$

$$\mathrm{S}xyz \longrightarrow xz(yz)$$

Turing complete



1924. "Über die Bausteine der mathematischen Logik"

Moses Schönfinkel

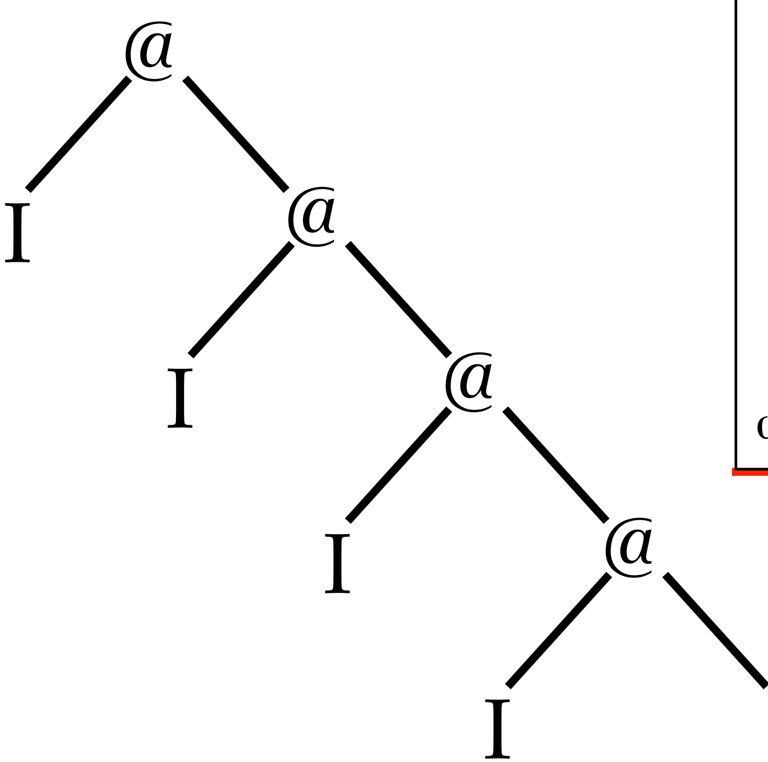
λ^∞ :not PML$^\infty$

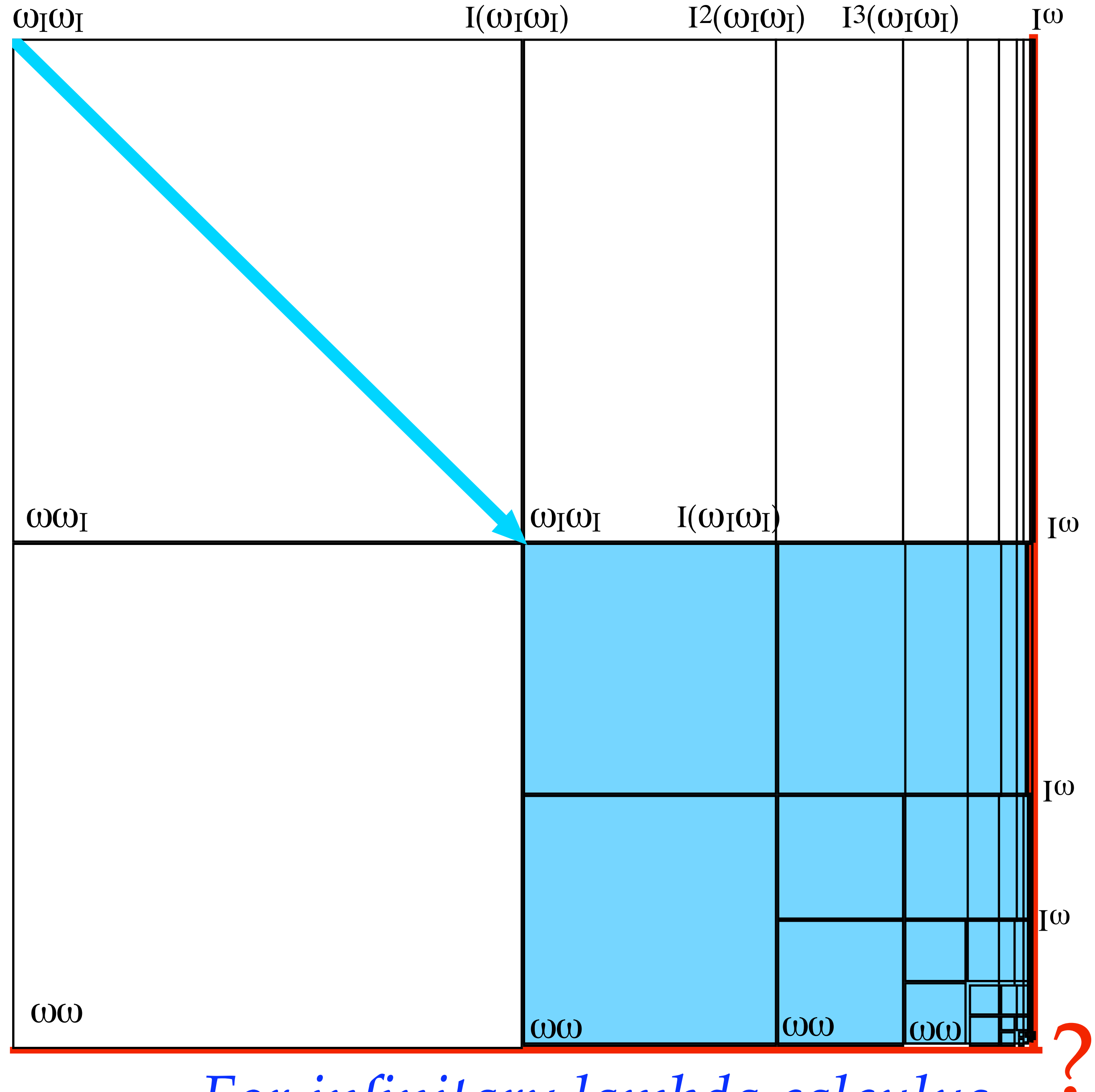
$$\omega_I \equiv (\lambda x.I(xx))$$

$$\omega \equiv \lambda x.xx$$

$$YI \to \omega_I\ \omega_I$$

I^ω $\equiv$





For infinitary lambda calculus Parallel Moves Lemma PML$^\infty$ fails, hence also CR$^\infty$

INFINITARY LAMBDA CALCULUS SUBSUMES SCOTT'S INDUCTION RULE

$$Yx \twoheadrightarrow x(Yx) \twoheadrightarrow x^2(Yx) \twoheadrightarrow^{\omega} x^{\omega} \equiv x(x(x(x\ldots$$

$$\begin{tikzcd}
BY \equiv (\lambda abc.a(bc))\, Y \arrow[dd] & \boxed{=_\infty} \quad \boxed{\neq_\beta} & BYS \equiv (\lambda abc.a(bc))\, YS \arrow[d, twoheadrightarrow] \\
 & & \lambda c.Y(Sc) \arrow[d, twoheadrightarrow] \\
\lambda bc.Y(bc) \arrow[ddd, "\omega"] & & \lambda c.\, Sc(Y(Sc)) \arrow[d, twoheadrightarrow] \\
 & & \lambda cz.\, cz(Y(Sc)z) \arrow[d, twoheadrightarrow] \\
 & & \lambda cz.\, cz(cz(Y(Sc)z)) \arrow[dll, "\omega"'] \\
\lambda bc.\, (bc)^{\omega} \equiv \lambda cz.\, (cz)^{\omega} & &
\end{tikzcd}$$

BY $\quad \boxed{\neq_\beta}$? $\quad$ BYS

BYI $\quad\quad$ BYSI

$\mathrm{BYI} \equiv (\lambda abc.a(bc))\ \mathrm{YI}$ $\quad\quad$ $\mathrm{BYSI} \equiv (\lambda abc.a(bc))\mathrm{YSI}$

$\downarrow$

$\downarrow$ $\quad\quad$ $\twoheaddownarrow$

$\lambda c.\mathrm{Y}(\mathrm{I}c)$

$\downarrow$

$\lambda c.\mathrm{Y}c$

$\twoheaddownarrow$

Y $\quad \boxed{\neq_\beta}$! $\quad$ Y(SI)

Curry's fpc $\quad\quad$ *Turing's fpc*

INFINITARY LAMBDA CALCULUS REPRESENTS AN ORACLE DIRECTLY

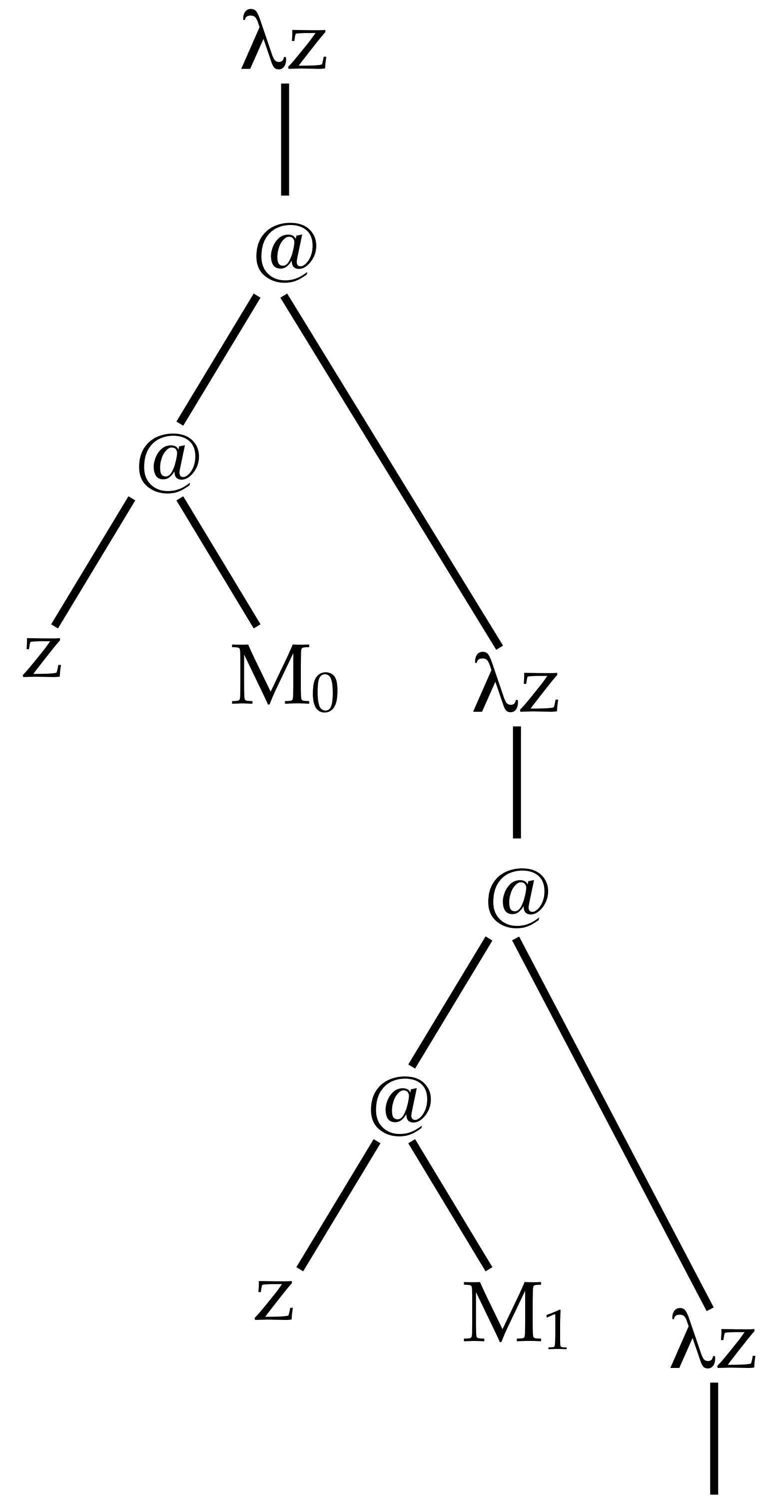
$[M,N] = \lambda z.zMN$

$[M_0 : M_1 : M_2 : \ldots] = [M_0, [M_1, [M_2, \ldots$

For $f: N \to N$, S_f

is $\underline{f(0)}:\underline{f(1)}:\underline{f(2)}: \ldots$

oracle

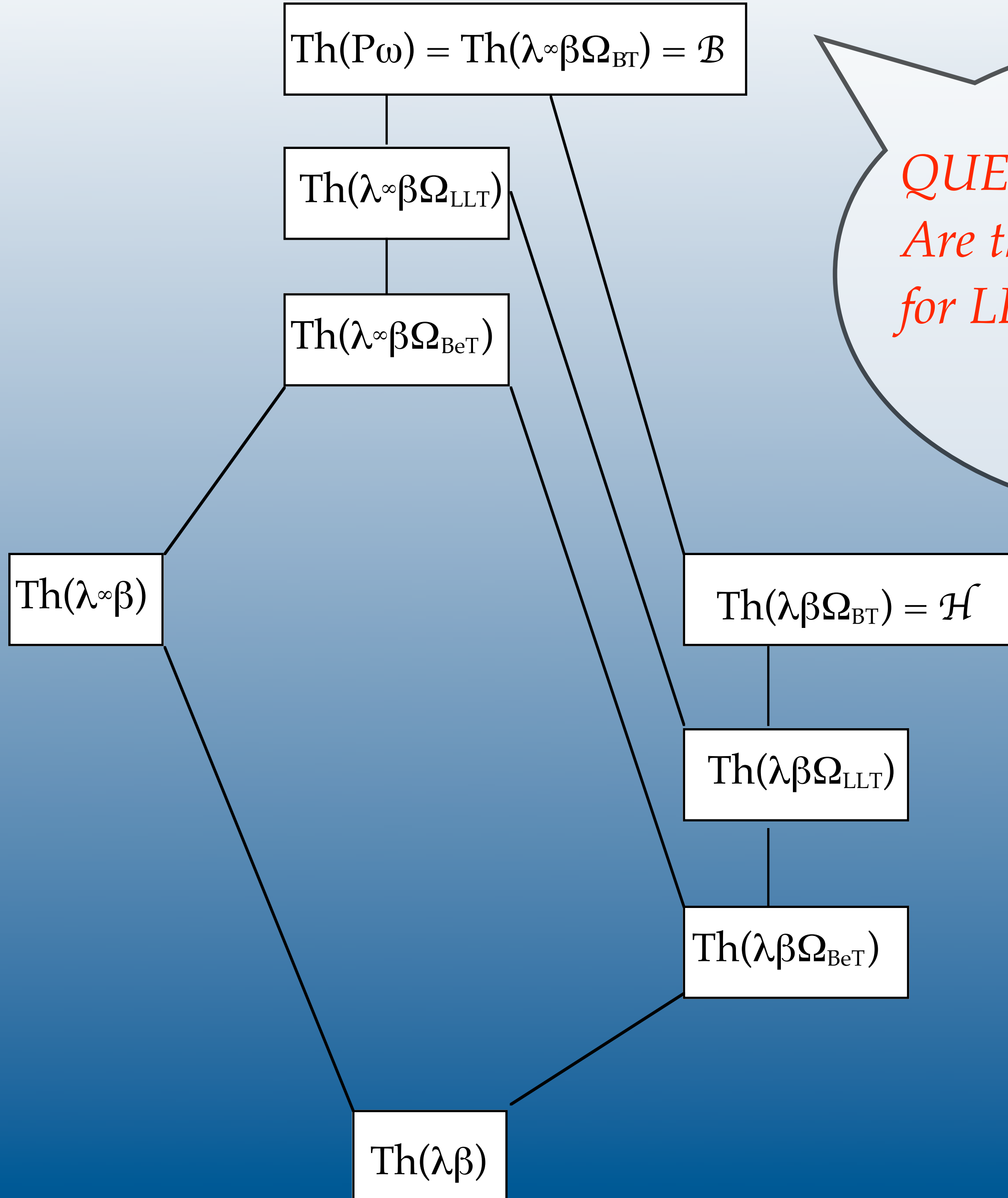


$g: N \to N$ *is recursive in* $f \Leftrightarrow$

for some finite λ*-term* T

$T\ S_f \ \to^{\omega}\ S_g$

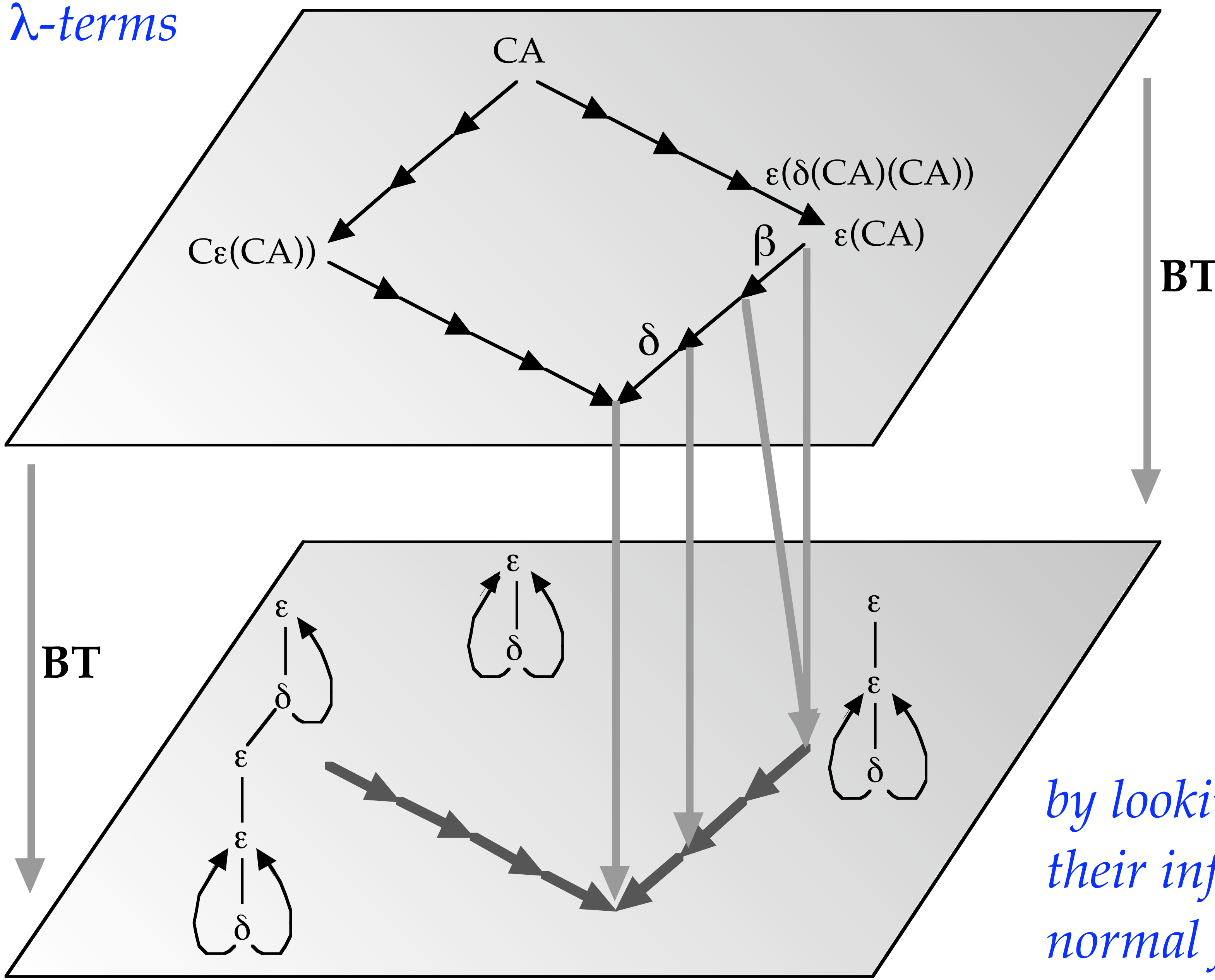
INFINITARY LAMBDA CALCULUS THEORIES



QUESTION:
Are there versions of $P\omega$ for LLT and BeT semantics?

INFINITARY LAMBDA CALCULUS WITH DXX -> X

explaining non-confluence for finite λ-terms



by looking at their infinite normal forms, the Böhm Trees

figure by joerg