

# Isabelle/HOL Exercises

## Lists

### Counting Occurrences

Define a function *occurs*, such that *occurs* *x* *xs* is the number of occurrences of the element *x* in the list *xs*.

```
consts occurs :: "'a ⇒ 'a list ⇒ nat"
```

Prove or disprove (by counterexample) the lemmas that follow. You may have to prove additional lemmas first. Use the *[simp]*-attribute only if the equation is truly a simplification and is necessary for some later proof.

```
lemma "occurs a xs = occurs a (rev xs)"
```

```
lemma "occurs a xs ≤ length xs"
```

Function *map* applies a function to all elements of a list: *map* *f* [*x*<sub>1</sub>, ..., *x*<sub>*n*</sub>] = [*f* *x*<sub>1</sub>, ..., *f* *x*<sub>*n*</sub>].

```
lemma "occurs a (map f xs) = occurs (f a) xs"
```

Function *filter* :: ('a ⇒ bool) ⇒ 'a list ⇒ 'a list is defined by

```
filter P [] = []
```

```
filter P (x # xs) = (if P x then x # filter P xs else filter P xs)
```

Find an expression *e* not containing *filter* such that the following becomes a true lemma, and prove it:

```
lemma "occurs a (filter P xs) = e"
```

With the help of *occurs*, define a function *remDups* that removes all duplicates from a list.

```
consts remDups :: "'a list ⇒ 'a list"
```

Find an expression *e* not containing *remDups* such that the following becomes a true lemma, and prove it:

```
lemma "occurs x (remDups xs) = e"
```

With the help of *occurs* define a function *unique*, such that *unique* *xs* is true iff every element in *xs* occurs only once.

```
consts unique :: "'a list ⇒ bool"
```

Show that the result of *remDups* is *unique*.