

Isabelle/HOL Exercises

Lists

Replace, Reverse and Delete

Define a function *replace*, such that *replace* *x* *y* *zs* yields *zs* with every occurrence of *x* replaced by *y*.

```
consts replace :: "'a ⇒ 'a ⇒ 'a list ⇒ 'a list"
```

```
primrec
```

```
"replace x y [] = []"
```

```
"replace x y (z#zs) = (if z=x then y else z)#(replace x y zs)"
```

Prove or disprove (by counterexample) the following theorems. You may have to prove some lemmas first.

```
lemma replace_append: "replace x y (xs @ ys) = replace x y xs @ replace x y ys"
```

```
  apply (induct "xs")
```

```
  apply auto
```

```
done
```

```
theorem "rev(replace x y zs) = replace x y (rev zs)"
```

```
  apply (induct "zs")
```

```
  apply (auto simp add: replace_append)
```

```
done
```

```
theorem "replace x y (replace u v zs) = replace u v (replace x y zs)"
```

```
  quickcheck
```

```
⋮
```

A possible counterexample: $u=0, v=1, x=0, y=-1, zs=[0]$

```
theorem "replace y z (replace x y zs) = replace x z zs"
```

```
  quickcheck
```

```
⋮
```

A possible counterexample: $x=1, y=0, z=1, zs=[0]$

Define two functions for removing elements from a list: *del1* *x* *xs* deletes the first occurrence (from the left) of *x* in *xs*, *delall* *x* *xs* all of them.

```
consts del1  :: "'a ⇒ 'a list ⇒ 'a list"
       delall :: "'a ⇒ 'a list ⇒ 'a list"
```

primrec

```
"del1 x []      = []"
"del1 x (y#ys) = (if y=x then ys else y # del1 x ys)"
```

primrec

```
"delall x []      = []"
"delall x (y#ys) = (if y=x then delall x ys else y # delall x ys)"
```

Prove or disprove (by counterexample) the following theorems.

```
theorem "del1 x (delall x xs) = delall x xs"
  apply (induct "xs")
  apply auto
done
```

```
theorem "delall x (delall x xs) = delall x xs"
  apply (induct "xs")
  apply auto
done
```

```
theorem delall_del1: "delall x (del1 x xs) = delall x xs"
  apply (induct "xs")
  apply auto
done
```

```
theorem "del1 x (del1 y zs) = del1 y (del1 x zs)"
  apply (induct "zs")
  apply auto
done
```

```
theorem "delall x (del1 y zs) = del1 y (delall x zs)"
  apply (induct "zs")
  apply (auto simp add: delall_del1)
done
```

```
theorem "delall x (delall y zs) = delall y (delall x zs)"
  apply (induct "zs")
  apply auto
done
```

```
theorem "del1 y (replace x y xs) = del1 x xs"
```

```

quickcheck
:
A possible counterexample: x=1, xs=[0], y=0
theorem "delall y (replace x y xs) = delall x xs"
  quickcheck
:
A possible counterexample: x=1, xs=[0], y=0
theorem "replace x y (delall x zs) = delall x zs"
  apply (induct "zs")
  apply auto
done

theorem "replace x y (delall z zs) = delall z (replace x y zs)"
  quickcheck
:
A possible counterexample: x=1, y=0, z=0, zs=[1]
theorem "rev(deli x xs) = deli x (rev xs)"
  quickcheck
:
A possible counterexample: x=1, xs=[1, 0, 1]
lemma delall_append: "delall x (xs @ ys) = delall x xs @ delall x ys"
  apply (induct "xs")
  apply auto
done

theorem "rev(delall x xs) = delall x (rev xs)"
  apply (induct "xs")
  apply (auto simp add: delall_append)
done

```