

Isabelle/HOL Exercises

Logic and Sets

Elimination of Connectives

In classical propositional logic, the connectives $=$, \vee , \neg can be replaced by \longrightarrow , \wedge , *False*. Define corresponding simplification rules as lemmas and prove their correctness. (You may use automated proof tactics.)

```
lemma equiv_conel: "(A = B) = ((A  $\longrightarrow$  B)  $\wedge$  (B  $\longrightarrow$  A))"
  by iprover
```

```
lemma or_conel: "(A  $\vee$  B) = ( $\neg$  ( $\neg$  A  $\wedge$   $\neg$  B))"
  by blast
```

```
lemma not_conel: "( $\neg$  A) = (A  $\longrightarrow$  False)"
  by blast
```

What is the result of your translation for the formula $A \vee (B \wedge C) = A$? (You can use Isabelle's simplifier to compute the result by using the simplifier's *only* option.)

Stating $A \vee (B \wedge C) = A$ as a lemma and application of
(*simp only: equiv_conel or_conel not_conel*)

results in the simplified goal

```
(A  $\longrightarrow$  False)  $\wedge$  ((B  $\wedge$  C  $\longrightarrow$  A)  $\wedge$  (A  $\longrightarrow$  B  $\wedge$  C)  $\longrightarrow$  False)  $\longrightarrow$  False.
```