Isabelle/HOL Exercises
Logic and Sets

A Riddle: Rich Grandfather

First prove the following formula, which is valid in classical predicate logic, informally with pen and paper. Use case distinctions and/or proof by contradiction.

If every poor man has a rich father,
then there is a rich man who has a rich grandfather.

theorem
```
"∀x. ¬ rich x ⟶ rich (father x) ⟹
∃x. rich (father (father x)) ∧ rich x"
```

Now prove the formula in Isabelle using a sequence of rule applications (i.e. only using the methods `rule`, `erule` and `assumption`).