Third Exam — Algorithm Theory, LVA 703608 — Institute of Computer Science
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Name: **Matr.-Nr.**: **Points**:

Good luck!

Big $\mathcal{O}$ notation (10 pts)

Which of the following functions are big-$\mathcal{O}$ of which others? (Recall that we are only interested in functions $f\colon \mathbb{N} \to \mathbb{N}$. Hence when we write $f(n)$, we really mean $\max(\{\lceil f(n) \rceil, 0\})$.) [5]

(a)
- $f_1(n) = \begin{cases} n^2 \cdot \log(n) & \text{if } n \text{ is even} \\ \frac{n^3}{\log(n)} & \text{if } n \text{ is odd} \end{cases}$
- n^n
- $n^2 \cdot \log(n)$
- $n^{5+\sin(n)}$
- 481^n

Here the answer suffices in the form of a diagram:

$$\begin{tikzcd}[column sep=small] & f_4 \arrow[dl, dash] \arrow[dr, dash] & \\ f_3 \arrow[dr, dash] & & f_2 \arrow[dl, dash] \\ & f_1 & \end{tikzcd}$$

(Meaning that $f_1(n) = \mathcal{O}(f_2(n))$, $f_1(n) = \mathcal{O}(f_3(n))$, $f_2(n) = \mathcal{O}(f_4(n))$ and $f_3(n) = \mathcal{O}(f_4(n))$ and neither $f_2(n) = \mathcal{O}(f_3(n))$ nor $f_3(n) = \mathcal{O}(f_2(n))$.)

(b) Explain your answers to (a) convincingly. [5]

Complexity Classes (14 pts)

(a) Define the complexity classes **PSPACE** and **NPSPACE**. [4]

(b) Prove the following assertion: **PSPACE** = **NPSPACE**. [5]

(c) Prove the following assertion: If **NP** $\neq$ **coNP**, then **P** $\neq$ **NP**. [5]

Hint: If you use a theorem to prove the second or third assertion, please give a brief explanation of how the theorem was proven or give the name of the proof method used.

Turing Machines (10 pts)

Consider the following Turing machine $\mathsf{M} = (K, \Sigma, \delta, s)$ with $K = \{s, s_a, s_b, s_c, s_d, s_e, yes, no\}$, $\Sigma = \{\mathsf{0}, \grave{\mathsf{0}}, \acute{\mathsf{0}}, \mathsf{1}, \grave{\mathsf{1}}, \acute{\mathsf{1}}, \triangleright, \sqcup\}$ and δ defined as follows:

$p \in K$	$\sigma \in \Sigma$	$\delta_M(p, \sigma)$
s	$\triangleright$	$(s_a, \triangleright, \rightarrow)$
s_a	$\mathsf{0}$	$(s_b, \grave{\mathsf{0}}, \rightarrow)$
s_a	$\mathsf{1}$	$(s_b, \grave{\mathsf{1}}, \rightarrow)$
s_a	$\acute{\mathsf{0}}$	$(s_e, \acute{\mathsf{0}}, \leftarrow)$
s_a	$\acute{\mathsf{1}}$	$(s_e, \acute{\mathsf{1}}, \leftarrow)$
s_b	$\mathsf{0}$	$(s_b, \mathsf{0}, \rightarrow)$
s_b	$\mathsf{1}$	$(s_b, \mathsf{1}, \rightarrow)$
s_b	$\sqcup$	$(s_c, \sqcup, \leftarrow)$
s_b	$\acute{\mathsf{0}}$	$(s_c, \acute{\mathsf{0}}, \leftarrow)$
s_b	$\acute{\mathsf{1}}$	$(s_c, \acute{\mathsf{1}}, \leftarrow)$

$p \in K$	$\sigma \in \Sigma$	$\delta_M(p, \sigma)$
s_c	$\mathsf{0}$	$(s_d, \acute{\mathsf{0}}, \leftarrow)$
s_c	$\mathsf{1}$	$(s_d, \acute{\mathsf{1}}, \leftarrow)$
s_c	$\grave{\mathsf{0}}$	$(\mathsf{no}, \grave{\mathsf{0}}, -)$
s_c	$\grave{\mathsf{1}}$	$(\mathsf{no}, \grave{\mathsf{1}}, -)$
s_d	$\mathsf{0}$	$(s_d, \mathsf{0}, \leftarrow)$
s_d	$\mathsf{1}$	$(s_d, \mathsf{1}, \leftarrow)$
s_d	$\grave{\mathsf{0}}$	$(s_a, \grave{\mathsf{0}}, \rightarrow)$
s_d	$\grave{\mathsf{1}}$	$(s_a, \grave{\mathsf{1}}, \rightarrow)$
s_e	$\grave{\mathsf{0}}$	$(s_e, \grave{\mathsf{0}}, \leftarrow)$
s_e	$\grave{\mathsf{1}}$	$(s_e, \grave{\mathsf{1}}, \leftarrow)$
s_e	$\triangleright$	$(\mathsf{yes}, \triangleright, -)$

(a) For arbitrary strings $x \in \{\mathsf{0}, \mathsf{1}\}^*$, what is $\mathsf{M}(x)$? [5]

(b) Explain informally how to extend M to decide the language $\mathrm{L} = \{ww \mid w \in \{\mathsf{0}, \mathsf{1}\}^*\}$. [5]

Reductions and NP-completeness (16 pts)

(a) Define the problems 3SAT and INDEPENDENT SET. [4]

(b) Define a reduction R from 3SAT to INDEPENDENT SET. [6]

(c) Prove that $\varphi \in$ 3SAT if and only if $R(\varphi) \in$ INDEPENDENT SET. [6]