

Name:

Matr.-Nr.:

Points:

*Good luck!*

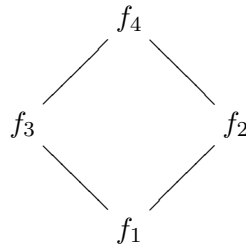
### Big $\mathcal{O}$ notation (10 pts)

Which of the following functions are big- $\mathcal{O}$  of which others? (Recall that we are only interested in functions  $f: \mathbb{N} \rightarrow \mathbb{N}$ . Hence when we write  $f(n)$ , we really mean  $\max(\{ \lceil f(n) \rceil, 0 \})$ .)

[5]

- (a) •  $f_1(n) = \begin{cases} n^2 \cdot \log(n) & \text{if } n \text{ is even} \\ \frac{n^3}{\log(n)} & \text{if } n \text{ is odd} \end{cases}$
- $n^n$
  - $n^2 \cdot \log(n)$
  - $n^{5+\sin(n)}$
  - $481^n$

Here the answer suffices in the form of a diagram:



(Meaning that  $f_1(n) = \mathcal{O}(f_2(n))$ ,  $f_1(n) = \mathcal{O}(f_3(n))$ ,  $f_2(n) = \mathcal{O}(f_4(n))$  and  $f_3(n) = \mathcal{O}(f_4(n))$  and neither  $f_2(n) = \mathcal{O}(f_3(n))$  nor  $f_3(n) = \mathcal{O}(f_2(n))$ .)

- (b) Explain your answers to (a) convincingly.

[5]

### Complexity Classes (14 pts)

- (a) Define the complexity classes **PSPACE** and **NPSPACE**. [4]
- (b) Prove the following assertion: **PSPACE** = **NPSPACE**. [5]
- (c) Prove the following assertion: If **NP**  $\neq$  **coNP**, then **P**  $\neq$  **NP**. [5]

*Hint:* If you use a theorem to prove the second or third assertion, please give a brief explanation of how the theorem was proven or give the name of the proof method used.

## Turing Machines (10 pts)

Consider the following Turing machine  $M = (K, \Sigma, \delta, s)$  with  $K = \{s, s_a, s_b, s_c, s_d, s_e, yes, no\}$ ,  $\Sigma = \{0, \dot{0}, \acute{0}, 1, \dot{1}, \acute{1}, \triangleright, \sqcup\}$  and  $\delta$  defined as follows:

$p \in K$	$\sigma \in \Sigma$	$\delta_M(p, \sigma)$	$p \in K$	$\sigma \in \Sigma$	$\delta_M(p, \sigma)$
$s$	$\triangleright$	$(s_a, \triangleright, \rightarrow)$	$s_c$	$0$	$(s_d, \acute{0}, \leftarrow)$
$s_a$	$0$	$(s_b, \dot{0}, \rightarrow)$	$s_c$	$1$	$(s_d, \acute{1}, \leftarrow)$
$s_a$	$1$	$(s_b, \dot{1}, \rightarrow)$	$s_c$	$\dot{0}$	$(no, \dot{0}, -)$
$s_a$	$\acute{0}$	$(s_e, \acute{0}, \leftarrow)$	$s_c$	$\dot{1}$	$(no, \dot{1}, -)$
$s_a$	$\acute{1}$	$(s_e, \acute{1}, \leftarrow)$	$s_d$	$0$	$(s_d, 0, \leftarrow)$
$s_b$	$0$	$(s_b, 0, \rightarrow)$	$s_d$	$1$	$(s_d, 1, \leftarrow)$
$s_b$	$1$	$(s_b, 1, \rightarrow)$	$s_d$	$\dot{0}$	$(s_a, \dot{0}, \rightarrow)$
$s_b$	$\sqcup$	$(s_c, \sqcup, \leftarrow)$	$s_d$	$\dot{1}$	$(s_a, \dot{1}, \rightarrow)$
$s_b$	$\acute{0}$	$(s_c, \acute{0}, \leftarrow)$	$s_e$	$\dot{0}$	$(s_e, \dot{0}, \leftarrow)$
$s_b$	$\acute{1}$	$(s_c, \acute{1}, \leftarrow)$	$s_e$	$\dot{1}$	$(s_e, \dot{1}, \leftarrow)$
			$s_e$	$\triangleright$	$(yes, \triangleright, -)$

- (a) For arbitrary strings  $x \in \{0, 1\}^*$ , what is  $M(x)$ ? [5]
- (b) Explain informally how to extend  $M$  to decide the language  $L = \{ww \mid w \in \{0, 1\}^*\}$ . [5]

## Reductions and NP-completeness (16 pts)

- (a) Define the problems 3SAT and INDEPENDENT SET. [4]
- (b) Define a reduction  $R$  from 3SAT to INDEPENDENT SET. [6]
- (c) Prove that  $\varphi \in 3SAT$  if and only if  $R(\varphi) \in INDEPENDENT SET$ . [6]