Name: **Matr.-Nr.**: **Points**:

Good luck!

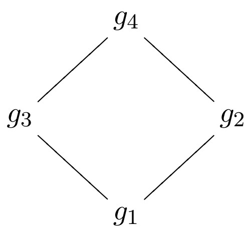
1 Small o-Notation (15 pts)

(a) Put the following functions in the correct order, i.e., put the largest function on the top line and put equivalent function on the same line.

$$2^n, n^2 \cdot \log(n), n^2, f(n), 3^n \ ,$$

Here $f(n) = \begin{cases} n^2 \cdot \sqrt{n} & n \text{ even} \\ 2^n & n \text{ odd} \end{cases}$

The answer suffices, for example in the form of a matrix or in the form of a diagram:



(Meaning that $g_1(n) = o(g_2(n))$, $g_1(n) = o(g_3(n))$, $g_2(n) = o(g_4(n))$ and $g_3(n) = o(g_4(n))$ and neither $g_2(n) = o(g_3(n))$ nor $g_3(n) = o(g_2(n))$.) [10]

(b) We write $f(n) \to \infty$ *for* $n \to \infty$ if $\forall C : \ \exists N : \ \forall n \geq N : \ f(n) > C$.
Prove the following implication:

$$g(n) = \mathcal{O}(h(n)) \Longrightarrow g(f(n)) = \mathcal{O}(h(f(n)))$$

if $f(n) \to \infty$ for $n \to \infty$. [5]

2 Complexity Classes (20 pts)

Definitions

(a) Give the definition of time constructible function. [5]

(b) Define the complexity classes **L** and **EXP**. [5]

Comparisons

Prove of disprove the below given relations between complexity classes. We write $\subseteq$ for set-theoretic inclusion (including equality) and $\subsetneq$ for strict inclusion. You may freely use any of the theorems in the slides or the book. In such a case sufficient information has to be given in order to understand which theorem is applied.

(c) $\mathbf{TIME}(n^2) \subsetneq \mathbf{TIME}(n^7)$. [5]

(d) $\mathbf{NSPACE}(n^2) \subseteq \mathbf{SPACE}(n^4)$. [5]

3 Turing Machines (15 pts)

Consider the Turing machine (TM) $\mathsf{M} = (K, \Sigma, \delta, s)$ for palindromes

$p \in K$	$\sigma \in \Sigma$	$\delta(p, \sigma)$
s	0	$(q_0, \triangleright, \rightarrow)$
s	1	$(q_1, \triangleright, \rightarrow)$
s	$\triangleright$	$(s, \triangleright, \rightarrow)$
s	$\sqcup$	$(yes, \sqcup, -)$
q_0	0	$(q_0, 0, \rightarrow)$
q_0	1	$(q_0, 1, \rightarrow)$
q_0	$\sqcup$	$(q_0', \sqcup, \leftarrow)$
q_1	0	$(q_1, 0, \rightarrow)$
q_1	1	$(q_1, 1, \rightarrow)$
q_1	$\sqcup$	$(q_1', \sqcup, \leftarrow)$

$p \in K$	$\sigma \in \Sigma$	$\delta(p, \sigma)$
q_0'	0	$(q, \sqcup, \leftarrow)$
q_0'	1	$(no, 1, -)$
q_0'	$\triangleright$	$(yes, \triangleright, \rightarrow)$
q_1'	0	$(no, 1, -)$
q_1'	1	$(q, \sqcup, \leftarrow)$
q_1'	$\triangleright$	$(yes, \triangleright, \rightarrow)$
q	0	$(q, 0, \leftarrow)$
q	1	$(q, 1, \leftarrow)$
q	$\triangleright$	$(s, \triangleright, \rightarrow)$

(a) Give good upper- and lower-bounds for the time-complexity of the TM M [7]

(b) Prove or disprove that for the language of palindromes PALINDROMES, we have PALINDROMES $\in \mathbf{SPACE}(\log n)$ [8]