

Name:

Matr.-Nr.:

Points:

Good luck!

## 1 Turing Machines (15 pts)

Consider the Turing machine (TM)  $M = (K, \Sigma, \delta, s)$  for palindromes

$p \in K$	$\sigma \in \Sigma$	$\delta(p, \sigma)$
$s$	0	$(q_0, \triangleright, \rightarrow)$
$s$	1	$(q_1, \triangleright, \rightarrow)$
$s$	$\triangleright$	$(s, \triangleright, \rightarrow)$
$s$	$\sqcup$	$(yes, \sqcup, -)$
$q_0$	0	$(q_0, 0, \rightarrow)$
$q_0$	1	$(q_0, 1, \rightarrow)$
$q_0$	$\sqcup$	$(q'_0, \sqcup, \leftarrow)$
$q_1$	0	$(q_1, 0, \rightarrow)$
$q_1$	1	$(q_1, 1, \rightarrow)$
$q_1$	$\sqcup$	$(q'_1, \sqcup, \leftarrow)$

$p \in K$	$\sigma \in \Sigma$	$\delta(p, \sigma)$
$q'_0$	0	$(q, \sqcup, \leftarrow)$
$q'_0$	1	$(no, 1, -)$
$q'_0$	$\triangleright$	$(yes, \triangleright, \rightarrow)$
$q'_1$	0	$(no, 1, -)$
$q'_1$	1	$(q, \sqcup, \leftarrow)$
$q'_1$	$\triangleright$	$(yes, \triangleright, \rightarrow)$
$q$	0	$(q, 0, \leftarrow)$
$q$	1	$(q, 1, \leftarrow)$
$q$	$\triangleright$	$(s, \triangleright, \rightarrow)$

- (a) Give good upper- and lower-bounds for the time-complexity of the TM  $M$  [7]
- (b) Prove or disprove that for the language of palindromes PALINDROMES, we have  $PALINDROMES \in \mathbf{NSPACE}(\log n)$  [8]

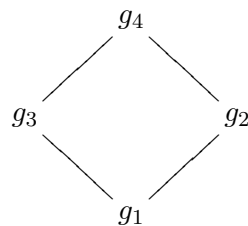
## 2 Small $o$ -Notation (15 pts)

- (a) Put the following functions in the correct order, i.e., put the largest function on the top line and put equivalent function on the same line.

$$2^{n \log n}, n^2 \cdot \sqrt{n}, n, f(n), 3^n,$$

$$\text{Here } f(n) = \begin{cases} n^2 \cdot \log(n) & n \text{ even} \\ 2^n & n \text{ odd} \end{cases}$$

The answer suffices, for example in the form of a matrix or in the form of a diagram:



(Meaning that  $g_1(n) = o(g_2(n))$ ,  $g_1(n) = o(g_3(n))$ ,  $g_2(n) = o(g_4(n))$  and  $g_3(n) = o(g_4(n))$  and neither  $g_2(n) = o(g_3(n))$  nor  $g_3(n) = o(g_2(n))$ .) [10]

- (b) We write  $f(n) \rightarrow \infty$  for  $n \rightarrow \infty$  if  $\forall C : \exists N : \forall n \geq N : f(n) > C$ .  
Prove the following implication:

$$g(n) = \mathcal{O}(h(n)) \implies g(f(n)) = \mathcal{O}(h(f(n)))$$

if  $f(n) \rightarrow \infty$  for  $n \rightarrow \infty$ . [5]

### 3 Complexity Classes (20 pts)

#### Comparisons

Prove or disprove the below given relations between complexity classes. We write  $\subseteq$  for set-theoretic inclusion (including equality) and  $\subsetneq$  for strict inclusion. You may freely use any of the theorems in the slides or the book. In such a case sufficient information has to be given in order to understand which theorem is applied.

- (a)  $\mathbf{TIME}(n) \subsetneq \mathbf{TIME}(n^4)$ . [5]  
(b)  $\mathbf{NSPACE}(n^3) \subseteq \mathbf{SPACE}(n^6)$ . [5]

#### Definitions

- (c) Give the definition of space constructible function. [5]  
(d) Define the complexity classes  $\mathbf{NL}$  and  $\mathbf{PSPACE}$ . [5]