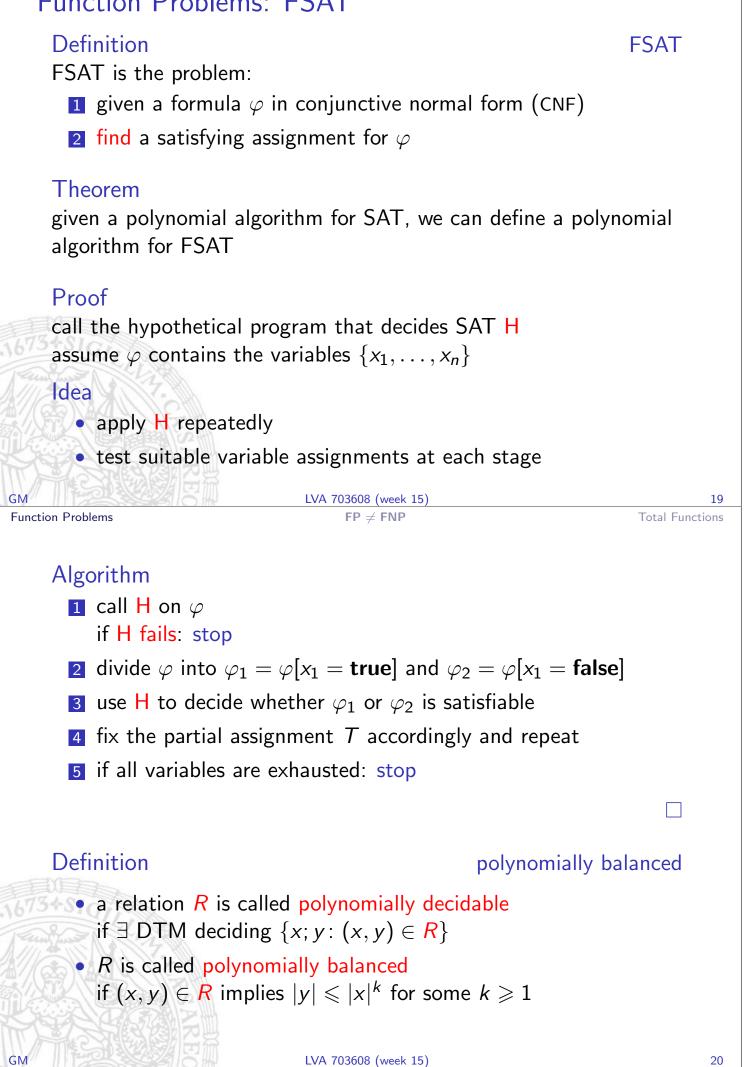
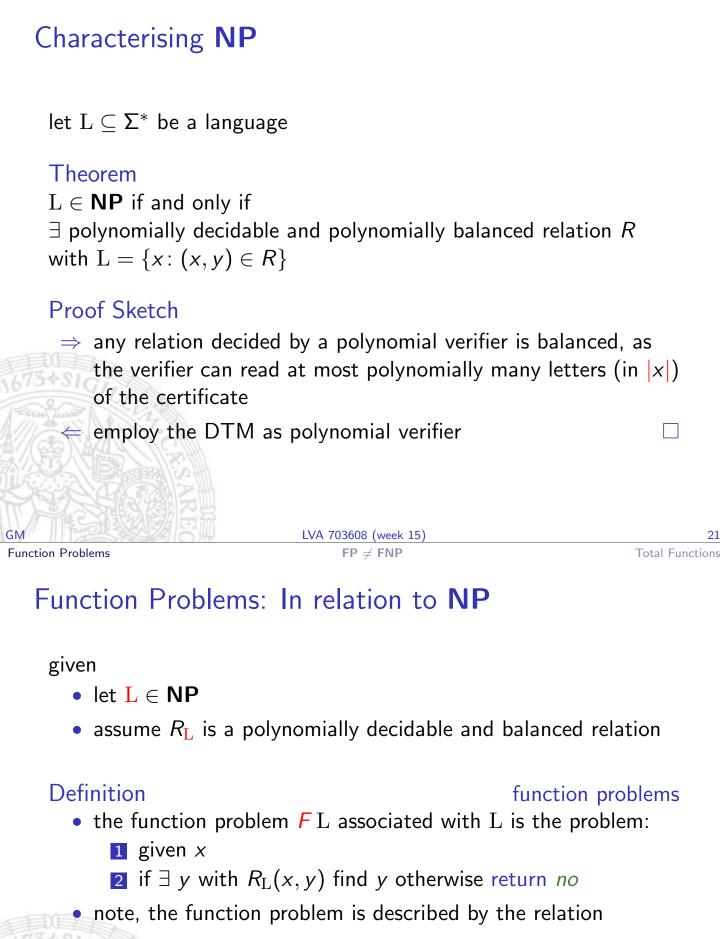


Function Problems: FSAT





Example

GM

FSAT is the function problem associated with SAT use the polynomially balanced relation for SAT

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Definition

- $\mathsf{FNP} = \{F L \mid L \in \mathsf{NP}\}$
- FP ⊆ FNP such that we only consider problems in FNP solvable in polytime

$\begin{array}{l} \text{Example} \\ \text{FSAT} \in \textbf{FNP} \end{array}$

Definition

logspace reduction

A reduces to B, if

- \exists functions $R: \Sigma^* \to \Sigma^*$ and $S: \Sigma^* \to \Sigma^*$
 - R, S logspace computable
- if x an instance of A then R(x) an instance of B
- if z is a correct output of B on R(x)
 then S(z) is a correct output of A on x

Function Problems

GM

GM

 $\frac{\text{LVA 703608 (week 15)}}{\text{FP} \neq \text{FNP}}$

Total Functions

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The $\mathbf{FP} \neq \mathbf{FNP}$ problem

DefinitionFNP-completenessa function problem A is complete for FNPif $A \in FNP$, and all problems in FNP reduce (in logspace) to A

Theorem FSAT is **FNP**-complete

Theorem FP = FNP iff P = NP.

Proof Sketch

 \Rightarrow by definition, i.e., the function problem is "stronger"

 $\Leftarrow \text{ assume } \mathbf{NP} = \mathbf{P}, \text{ in particular } \mathsf{SAT} \in \mathbf{P} \\ \text{ as FSAT is } \mathbf{FNP}\text{-complete, we only need to show } \mathsf{FSAT} \in \mathbf{FP} \\ \text{ however } \mathsf{SAT} \in \mathbf{P} \text{ implies } \mathsf{FSAT} \in \mathbf{FP} \\ \Box$

$\mathbf{FP}\neq\mathbf{FNP}$

Total Functions

Definition total function • a problem R in **FNP** is total if \forall strings x \exists at least one string y such that R(x, y) this subclass is denoted as TFNP Remark total function problems correspond to the language $L = \Sigma^*$ hence the corresponding decision problem is meaningless Example FACTORING 1 given an integer n **2** find its prime decomposition $n = p_1^{k_1} \cdots p_m^{k_m}$ together with primality certificates for p_1, \ldots, p_m GM LVA 703608 (week 15) 25