



3 June 2008

## Proseminar Algorithmen und Datenstrukturen

# Exercise Sheet 10

### Exercise 1 (Master Theorem)

Give a recurrence equation for the worst-case running time and a tight asymptotic ( $\Theta$ -notation) bound on the worst-case running time of the following algorithms:

- a) Binary search in a sorted array of size  $n$ .
- b)  $\text{StoogeSort}(A, 1, n)$  where  $n$  is the size of the array  $A$ .

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#### Listing 1 *StoogeSort*

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**Input:** array  $A$ , start index  $i$ , stop index  $j$

**Output:** the array elements  $A[i..j]$  are sorted!

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1: if  $A[i] > A[j]$  then
2:   exchange  $A[i] \leftrightarrow A[j]$ 
3: if  $i + 1 \geq j$  then
4:   return
5:  $k := \lfloor (j - i + 1)/3 \rfloor$ 
6:  $\text{StoogeSort}(A, i, j - k)$ 
7:  $\text{StoogeSort}(A, i + k, j)$ 
8:  $\text{StoogeSort}(A, i, j - k)$ 
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### Exercise 2 (Stooge Sort)

Prove that  $\text{StoogeSort}(A, 1, \text{length}(A))$  correctly sorts the input array  $A$  by induction on the length of  $A$ .

### Exercise 3 (Quicksort)

Construct a non-trivial example for which quicksort will use  $\Omega(n^2)$  comparisons when the pivot is chosen by taking the median of the first, last and middle elements of the sequence ( the median of a finite list of numbers can be found by arranging them from lowest value to highest value and picking the middle one, e.g.  $median(4, 3, 5) = 4$  ).

### Exercise 4 (Sorting Algorithms)

- a) Implement *mergesort* in C and incorporate it into the framework of Exercise 3 of last week.
- b) Implement *quicksort* in C and incorporate it into the framework of Exercise 3 of last week.