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SS 2008
Exercise Sheet 3
Due 8 May 2008

Automatic Deduction — Introduction to Isabelle

LVA 703522

1 Isar

1.1 Propositional Logic

▷ Provide Isar proof texts that prove the following lemmas. You may only use the methods `rule` and `-`.

```
lemma "(A ∨ A) = (A ∧ A)"
proof
  assume "A ∨ A"
  show "A ∧ A" sorry
next
  assume "A ∧ A"
  show "A ∨ A" sorry
qed
```

```
lemma "(A ∨ B) ∨ C ⟶ A ∨ (B ∨ C)"
oops
```

1.2 Predicate Logic

▷ Provide Isar proof texts that prove the following lemmas. Again, you may not use automation.

```
lemma "((∀ x. P x) ∧ (∀ x. Q x)) = (∀ x. (P x ∧ Q x))"
oops
```

```
lemma "((∃ x. P x) ∨ (∃ x. Q x)) = (∃ x. (P x ∨ Q x))"
oops
```

The following lemma also requires `classical: (¬ P ⟹ P) ⟹ P` (or an equivalent theorem) in order to be proved. You need to invoke this explicitly with **proof** `rule classical` or similar.

```
lemma "(¬ (∀ x. P x)) = (∃ x. ¬ P x)"
oops
```

Hint: it may be useful to study the natural deduction proofs of these lemmas before attempting to provide Isar proofs.

2 Rich Grandfather

Recall the "Rich Grandfather" riddle (Exercises 2).

If every poor man has a rich father,
then there is a rich man who has a rich grandfather.

▷ Give an Isar proof of the theorem. You may use automated tactics, but the general proof structure should resemble your informal pen-and-paper proof of the theorem.

```
theorem
  "∀x. ¬ rich x ⟶ rich (father x) ⟹
  ∃x. rich (father (father x)) ∧ rich x"
oops
```