First Exam
Complexity Theory

July 4, 2008

Name: **Studentnumber**:

The exam consists of 5 exercises with a total of 50 points.

1	2	3	4	5	Sum

1. Consider one-tape Turing machines that are constrained not to overwrite the input string. They may write all they want on the blank portion of the tape to the right. Show that these machines accept only regular sets. (10 pts)

   Hint: Use the following fact: Let $R \subseteq \Sigma^*$ be a set. Given a relation $\equiv$ that fulfills (i) $x \equiv y$ implies $xa \equiv ya$, (ii) $x \equiv y$ implies $x \in R \Leftrightarrow y \in R$, and (iii) $\equiv$ has only finitely many equivalence classes. Then R is regular.

2. Consider Savitch's Theorem.

   a) Precisely state this theorem with all necessary assumptions and give (at least) one crucial consequence of the theorem. (4 pts)

   b) Sketch the proof of the theorem. (8 pts)

3. Consider oracle Turing machines and assume the oracle is described as a set B.

   a) Formally define the complexity classes P^B and NP^B. (4 pts)

   b) Prove the existence of an oracle B such that $\mathsf{P}^B = \mathsf{NP}^B$. (8 pts)

4. Show that if $\mathsf{NP} = \mathsf{coNP}$, then the Polynomial Hierarchy collapses to NP. (6 pts)

5. Determine whether the following statements are true or false. Every correct answer is worth 2 points (and every wrong 0 points). (10 pts)

statement	**yes**	**no**
Deciding whether $L(M) = \Sigma^*$ for a given DFA M is $\leqslant_m^{\log}$-hard for PSPACE	☐	☐
$\mathsf{MAZE} \in \mathsf{LOGSPACE}$ if and only if $\mathsf{LOGSPACE} = \mathsf{NLOGSPACE}$	☐	☐
$\mathsf{NSPACE}(\log^2(n)) \subseteq co - \mathsf{NSPACE}(\log^4(n))$	☐	☐
$\mathsf{NSPACE}(\log^4(n)) \subseteq \mathsf{ATIME}(\log^{4711}(n))$	☐	☐
$A \in \Sigma_2^{\mathrm{p}}$ if and only if $A = \{x \mid \exists y_1 \forall y_2 \ldots \mathsf{Q} y_k R(x, y_1, \ldots, y_k)$, for a deterministic polytime computable $(k+1)$-ary predicate R and $\mathsf{Q} \in \{\exists, \forall\}$	☐	☐