

Second Exam Complexity Theory

September 29, 2008

Name:

Studentnumber:

The exam consists of 5 exercises with a total of 50 points.

1	2	3	4	5	Sum
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1. Provide a non-regular set that is accepted in $O(\log \log n)$ space by a deterministic TM. (It suffices to give the set and sketch the argument why this language is in $DSPACE(\log \log n)$.) (8 pts)

2. Consider Immerman-Szelepcsényi Theorem.
 - a) Precisely state this theorem with all necessary assumptions and give (at least) one crucial consequence of the theorem. (4 pts)
 - b) Sketch the proof of the theorem. (8 pts)

3. Consider alternating Turing machines
 - a) Formally define the complexity classes **ALOGSPACE**, **APTIME**, **APSPACE**, **AEXPTIME**. (This includes of course a formal definition of ATMs.) (6 pts)
 - b) Show that an ATM without negations accepts x if and only if there is a finite accepting subtree of the computation tree on input x ; that is a finite subtree T of the computation tree containing the start configuration such that every \vee -configuration has at least one successor and every \wedge -configuration has all its successors in t . (8 pts)

4. Consider **PSPACE**-complete problems.
 - a) Formally define the quantified Boolean formula problem (QBF). (2 pts)
 - b) Show that QBF is \leq_m^{\log} -complete for **PSPACE**. (It suffices to sketch the arguments.) (6 pts)

Hint: To prove that QBF is \leq_m^{\log} -hardness for **PSPACE**, we used a clever connection between ATMs and DTMs.

5. Determine whether the following statements are true or false. Suppose that $T(n) \geq n$ and $S(n) \geq \log n$. Every correct answer is worth 2 points (and every wrong 0 points). (8 pts)

statement	true	false
$ATIME(T(n)) \subsetneq DSPACE(T(n))$	<input type="checkbox"/>	<input type="checkbox"/>
$DSPACE(S(n)) \subseteq ATIME(S(n)^2)$	<input type="checkbox"/>	<input type="checkbox"/>
$ASPACE(S(n)) \subseteq DTIME(O(S(n)))$	<input type="checkbox"/>	<input type="checkbox"/>
$DTIME(T(n)) \subseteq ASPACE(T(n))$	<input type="checkbox"/>	<input type="checkbox"/>