

Third Exam Complexity Theory

March 6, 2009

Name:

Studentnumber:

The exam consists of 6 exercises with a total of 50 points.

1	2	3	4	5	6	Sum
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1. Consider Turing machines.
 - a) Prove the following assertion: “Let $\Sigma = \{0, 1, \#\}$. The set of palindromes $\text{PAL} := \{z \in \Sigma^* \mid z = \text{rev } z\}$ requires $\Omega(n^2)$ time on a one-tape TM”. (8 pts)
Hint: Consider the following subset of all palindromes:
$$\text{PAL}_n = \{x \#^{\frac{n}{2}} \text{rev } x \mid x \in \{0, 1\}^{\frac{n}{4}}\}.$$
And use a crossing sequence argument for each element $x \in \text{PAL}_n$.
 - b) Does the assertion in point 1.a) hold for multi-tape TMs as well? Explain your answer. (3 pts)
2. Assume $S: \mathbb{N} \rightarrow \mathbb{N}$ such that $S(n) \geq \log n$. Show that
$$\text{NSPACE}(S(n)) \subseteq \text{DTIME}(2^{O(S(n))}).$$
 (6 pts)
3. Consider logspace computability.
 - a) Give a formal definition of logspace transducers and logspace computability. (4 pts)
 - b) Give a formal definition of the problem MAZE. (2 pts)
 - c) Show the following assertion “MAZE is \leq_m^{\log} -complete for NLOGSPACE”. (5 pts)
4. Consider oracle TMs and assume the oracle is described as a set B .
 - a) Formally define the polynomial hierarchy in terms of oracle TMs (2 pts)
 - b) Prove the existence of an oracle B such that $\text{P}^B = \text{NP}^B$. (6 pts)
5. Give a complete definition of Nick’s class (NC). (4 pts)
6. Determine whether the following statements are true or false. (10 pts)
Every correct answer is worth 2 points (and every wrong -1 points).

statement	yes	no
$\text{NSPACE}(n^3) \subsetneq \text{NSPACE}(n^7)$.	<input type="checkbox"/>	<input type="checkbox"/>
MAZE \in LOGSPACE only if LOGSPACE = NLOGSPACE.	<input type="checkbox"/>	<input type="checkbox"/>
Generalised geography is \leq_m^{\log} -complete for PSPACE.	<input type="checkbox"/>	<input type="checkbox"/>
$\text{NC} = \text{STA}(\log n, n^{O(1)}, (\log n)^{O(1)})$.	<input type="checkbox"/>	<input type="checkbox"/>
$\text{RP} \subseteq \text{BPP}$, but BPP is not closed under complement.	<input type="checkbox"/>	<input type="checkbox"/>