

# Third Exam Complexity Theory

March 6, 2009

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**Name:**

**Studentnumber:**

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The exam consists of 6 exercises with a total of 50 points.

1	2	3	4	5	6	Sum
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1. Consider Turing machines.
- a) Prove the following assertion: “Let  $\Sigma = \{0, 1, \#\}$ . The set of palindromes  $\text{PAL} := \{z \in \Sigma^* \mid z = \text{rev } z\}$  requires  $\Omega(n^2)$  time on a one-tape TM”. (8 pts)  
*Hint:* Consider the following subset of all palindromes:

$$\text{PAL}_n = \{x \#^{\frac{n}{2}} \text{rev } x \mid x \in \{0, 1\}^{\frac{n}{4}}\}.$$

And use a crossing sequence argument for each element  $x \in \text{PAL}_n$ .

- b) Does the assertion in point 1.a) hold for multi-tape TMs as well? Explain your answer. (3 pts)
2. Assume  $S: \mathbb{N} \rightarrow \mathbb{N}$  such that  $S(n) \geq \log n$ . Show that

$$\text{NSPACE}(S(n)) \subseteq \text{DTIME}(2^{O(S(n))}).$$

(6 pts)

3. Consider logspace computability.
- a) Give a formal definition of logspace transducers and logspace computability. (4 pts)  
b) Give a formal definition of the problem MAZE. (2 pts)  
c) Show the following assertion “MAZE is  $\leq_m^{\log}$ -complete for NLOGSPACE”. (5 pts)

4. Consider oracle TMs and assume the oracle is described as a set  $B$ .
- a) Formally define the polynomial hierarchy in terms of oracle TMs (2 pts)  
b) Prove the existence of an oracle  $B$  such that  $\text{P}^B = \text{NP}^B$ . (6 pts)

5. Give a complete definition of Nick’s class (NC). (4 pts)

6. Determine whether the following statements are true or false. (10 pts)  
Every correct answer is worth 2 points (and every wrong -1 points).

statement	yes	no
$\text{NSPACE}(n^3) \subsetneq \text{NSPACE}(n^7)$ .	<input type="checkbox"/>	<input type="checkbox"/>
$\text{MAZE} \in \text{LOGSPACE}$ only if $\text{LOGSPACE} = \text{NLOGSPACE}$ .	<input type="checkbox"/>	<input type="checkbox"/>
Generalised geography is $\leq_m^{\log}$ -complete for PSPACE.	<input type="checkbox"/>	<input type="checkbox"/>
$\text{NC} = \text{STA}(\log n, n^{O(1)}, (\log n)^{O(1)})$ .	<input type="checkbox"/>	<input type="checkbox"/>
$\text{RP} \subseteq \text{BPP}$ , but BPP is not closed under complement.	<input type="checkbox"/>	<input type="checkbox"/>