

Complexity Theory

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10/1

	Homework
<ul> <li>Definition generalised geography game if</li> <li>(C, E) directed graph</li> <li>s ∈ C</li> <li>Player I starts in s = s<sub>0</sub> and moves in even states from s<sub>2i</sub> to adjacent vertex s<sub>2i+1</sub></li> <li>Player II moves from s<sub>2i+1</sub> to s<sub>2i+2</sub></li> <li>vertexes must not be revisited</li> </ul> Theorem Generalised geography is ≤ <sup>log</sup> <sub>m</sub> -complete for PSPACE	<ol> <li>Miscellaneous Exercises 11</li> <li>Miscellaneous Exercises 14</li> <li>Miscellaneous Exercises 26</li> </ol>
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Definition of PH via ATMS Definition of PH via ATMS $D_{k}$ -machine a $\Sigma_{k}$ -machine is an ATM for which the computation path is dividable in separate sections on any input and any section consists only of $\wedge$ - or $\vee$ -configurations a t most k sections b the first consist of $\vee$ -configurations a $\Pi_{k}$ -machine is defined by swapping $\vee$ and $\wedge$ $\Sigma_{0}$ , $\Pi_{0}$ are defined to be deterministic TMs Example a $\Sigma_{1}$ -machine is a nondeterministic TM Definition $\Sigma_{k}^{p} := \{L(M) \mid M \text{ is polytime bounded } \Sigma_{k}^{p}-\text{machines} \}$ $\Pi_{k}^{p} := \{L(M) \mid M \text{ is polytime bounded } \Pi_{k}^{p}-\text{machines} \}$	Lemma $\Pi_{k}^{p} = co - \Sigma_{k}^{p} = \{\sim A \mid A \in \Sigma_{k}^{p}\}$ $\Sigma_{k}^{p} \cup \Pi_{k}^{p} \subseteq \Sigma_{k+1}^{p} \cap \Pi_{k+1}^{p}$ $\bigcup_{k \ge 1} \Sigma_{k}^{p} = \bigcup_{k \ge 0} \Pi_{k}^{p} \subseteq PSPACE$ Definition $H_{k} := \{M \# x \#^{m} \mid M \text{ an ATM and } M_{k}^{m} \text{ accepts } x\}$ here $M_{k}^{m}$ denotes the modification of $M$ such that 1 at most $k$ intervals of $\wedge$ - and $\vee$ -configurations, beginning with $\vee$ 2 runtime at most $m$ Lemma $H_{k} := \{orrestance for \Sigma_{k}^{p} \text{ and } \sim H_{k} \text{ is } \leq_{m}^{\log}\text{-complete for } \Pi_{k}^{p}$

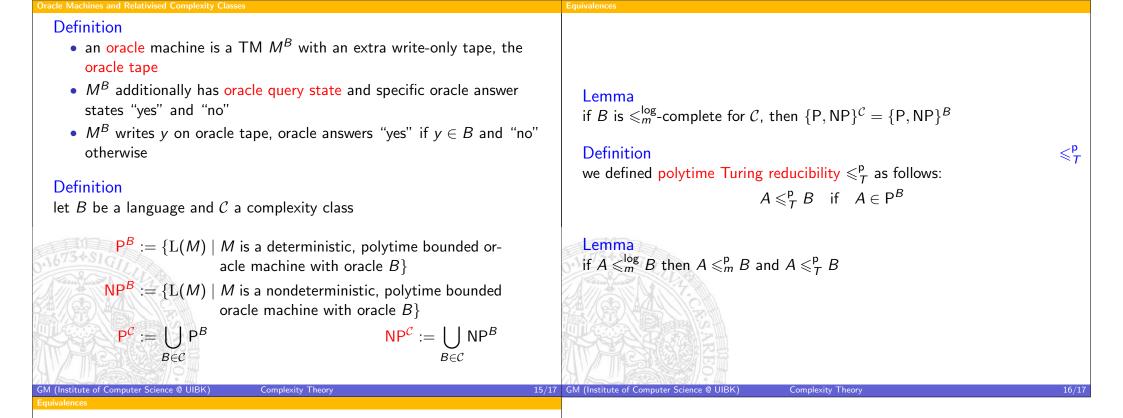
13/17 GM (Institute of Computer Science @ UIBK)

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14/17

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## Theorem

consider

$$\mathsf{NP} \subseteq \mathsf{NP}^{\mathsf{NP}} \subseteq \mathsf{NP}^{\mathsf{NP}^{\mathsf{NP}}} \dots$$

i.e.,  $NP_1 := NP$  and  $NP_{k+1} := NP^{NP_k}$ , then  $\forall k \ge 1$ :  $NP_k = \Sigma_k^p$ 

define  $\exists^t x \ \varphi(x) :\Leftrightarrow \exists x | y | \leq t \land \varphi(x)$  and  $\forall^t x \ \varphi(x) :\Leftrightarrow \forall x | y | \leq t \rightarrow \varphi(x)$ 

## Theorem

a language L is in  $\Sigma_k^p$  iff there is a deterministic polytime computable (k+1)-ary predicate R and a constant c such that

$$A = \{x \mid \exists^{|x|^c} y_1 \forall^{|x|^c} y_2 \exists^{|x|^c} y_3 \dots Q^{|x|^c} y_k R(x, y_1, \dots, y_k)\}$$

 $(\mathsf{Q} \in \{\exists,\forall\})$ 

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17/1