## First name:

## Last name:

$\qquad$
Matriculation number:

- Write your name and matriculation number on every page.
- Please answer all exercises in a readable and precise way. Please cross out solution attempts which are replaced by another solution.
- Cheating is not allowed. Everyone who is caught will fail the exam.
- Please do not remove the staples of the exam.

| Exercise | Maximal points | Points |
| :--- | :---: | :---: |
| 1 | 35 |  |
| 2 | 30 |  |
| 3 | 20 |  |
| 4 | 15 |  |
| $\Sigma$ | 100 |  |
| Grade |  |  |

## Exercise $1(20+15$ points $)$

Consider the following transition system and formula.


- Complete the game-graph which is depicted on the next page.
- Color the game-graph using the bottom-up coloring algorithm. Mark your nodes by (g)reen or (r)ed and additionally indicate whether a node was colored during a (p)ropagation-phase or whether it was colored since it remained (w)hite after propagation stopped. So, label each node with one of $\{\mathbf{g p}, \mathbf{r p}, \mathbf{g w}, \mathbf{r w}\}$.



## Exercise $2(18+12$ points $)$

Consider the following NBA $\mathcal{A}$.


- Compute the $\mathcal{A}$-equivalence classes by giving their shortest representatives and the corresponding transition profiles.

| representative $w$ | $\operatorname{tp}(\mathrm{w})$ |
| :--- | :--- |
|  |  |
|  |  |
|  |  |
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|  |  |
|  |  |
|  |  |
|  |  |

- Construct the S1S-formula $\varphi_{\mathcal{A}}(\mathrm{a})$ using the construction from the lecture (and not the optimized version from the exercises).


## Exercise 3 (20 points)

Consider a crossing with two traffic lights, each having phases red-orange-green-orange-red-.... Construct timed automata $T A_{i}(i \in\{1,2\})$ for each of the traffic lights such that the following conditions are satisfied:

- The combination of the lights is safe, i.e., if one light is in a non-red state, then the other light shows red.
- Each green phase is between 30 and 40 seconds long.
- There is a delay between 2 and 3 seconds after the switch from orange to red on the one light, before the switch from red to orange is performed on the other light.
- Each orange phase takes exactly 5 seconds.
- Both lights show green infinitely often.
- $T A_{1}$ and $T A_{2}$ are symmetrical, only the initial state differs: $T A_{1}$ starts in a red state, $T A_{2}$ in a green one.

For symmetry reasons you only have write down $T A_{1}$.

## Exercise 4 (15 points)

The theorem of Knaster \& Tarski can be lifted to infinite sets $S$. Essentially, one replaces $\tau^{|S|}(\varnothing)$ by

$$
f p_{\tau}=\bigcup_{n \in \mathbb{N}} \tau^{n}(\varnothing)
$$

Prove that if $\tau: 2^{S} \rightarrow 2^{S}$ is monotone then $f p_{\tau} \subseteq \tau\left(f p_{\tau}\right)$.

