Model Checking

René Thiemann

Institute of Computer Science
University of Innsbruck

SS 2008

Outline

- Why Real-Time Systems are Important
- Timed Automata - A Way to Model Real-Time Systems
  - Syntax and Composition
  - Semantic
  - Undesired Behaviors
- Timed CTL
- Model Checking for Timed CTL
  - Region Transition System
  - Model Checking via CTL
- Summary

Examples

Clocks and Constraints

Clocks

- A clock can measure times $\in \mathbb{R}^{\geqslant 0}$
- Clocks are usually written by $x, y, \ldots$, sets of clocks are $C, D, \ldots$

Clock-Constraints

A clock-constraint over clocks C is $g \in CC(C)$:

$$g ::= x < c \mid x \leqslant c \mid x > c \mid x \geqslant c \mid g \wedge g$$

where $x \in C, c \in \mathbb{N}$

- Extension to rational numbers possible (but simple to avoid)
- Constraints of form $x - y < c, \ldots$ possible (but not considered)

Timed Automata

Main ideas:

- Global time
- Add clock constraints to states (invariants) and transitions (guards)
- Clocks can be reseted when performing transition
- Time can elapse in states
- Transitions are performed instantaneous
- Parallel composition of timed automata via hand-shaking actions

---

Timed Automata (formally)

A timed automata is octuple $TA = (Loc, Act, C, \longrightarrow, Loc_0, Inv, AP, L)$

- Loc: set of locations
- Act: set of actions
- Loc_0: set of initial locations
- AP: set of atomic propositions
- L: labeling function, $L : Loc \rightarrow 2^{AP}$
- C: set of clocks
- $\longrightarrow$: transition relation, $\longrightarrow \subseteq Loc \times CC(C) \times Act \times 2^{C} \times Loc$
- Inv: invariant assignment, $Inv : Loc \rightarrow CC(C)$

State of transition system for timed automaton consists of location and clock-evaluation α ($\alpha : C \rightarrow \mathbb{R}^{\geqslant 0}$)

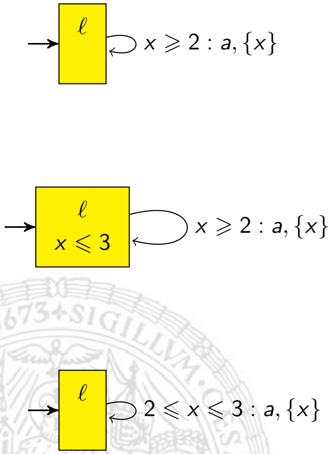
Meaning of $s \xrightarrow{g:a,D} t$: If α satisfies g then one can perform a-step and all clocks in D will be reseted to 0.

Meaning of $Inv(s) = g$: One can only stay in s if α satisfies g

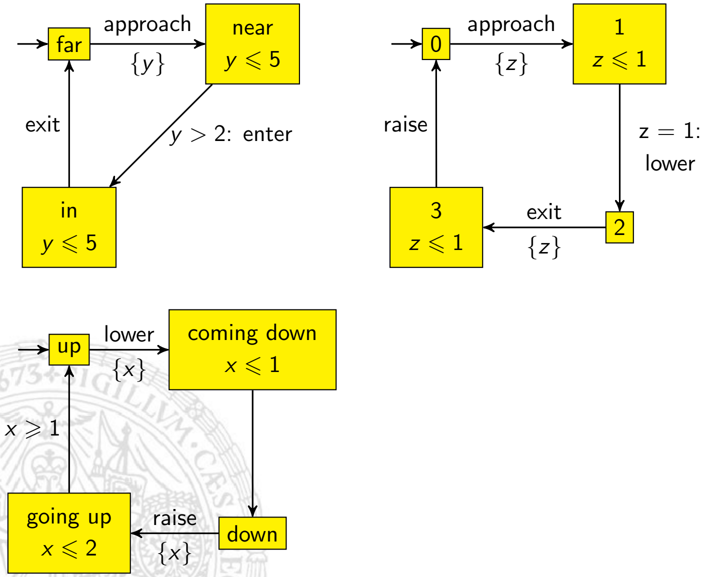
---

Example: Guards versus Location Invariants

Convention: Omit constraint "true" and empty set of reseted clocks



Example: Train-Gate-Controller



Composing Timed Automata

Definition

Given $TA_i = (Loc_i, Act_i, C_i, \rightarrow_i, Loc_{0,i}, Inv_i, AP_i, L_i)$ where $AP_1 \cap AP_2 = C_1 \cap C_2 = \varnothing$. Let $H \subseteq Act_1 \cap Act_2$ be a set of handshake-actions. Then the timed automaton $TA_1 ||_H TA_2$ is defined as $(Loc_1 \times Loc_2, Act_1 \cup Act_2, C_1 \cup C_2, \rightarrow, Loc_{0,1} \times Loc_{0,2}, Inv, AP_1 \cup AP_2, L)$

- $L((\ell_1, \ell_2)) = L_1(\ell_1) \cup L_2(\ell_2)$
- $Inv((\ell_1, \ell_2)) = Inv_1(\ell_1) \wedge Inv_2(\ell_2)$
- $\rightarrow$ is defined as follows:

$$\frac{\ell_1 \xrightarrow{g_1:a,D_1}{}_1 \ell_1' \qquad \ell_2 \xrightarrow{g_2:a,D_2}{}_2 \ell_2'}{(\ell_1, \ell_2) \xrightarrow{g_1 \wedge g_2:a,D_1 \cup D_2} (\ell_1', \ell_2')} \text{ if } a \in H$$

$$\frac{\ell_1 \xrightarrow{g_1:a,D_1}{}_1 \ell_1'}{(\ell_1, \ell_2) \xrightarrow{g_1:a,D_1} (\ell_1', \ell_2)} \text{ if } a \notin H \qquad \frac{\ell_2 \xrightarrow{g_2:a,D_2}{}_2 \ell_2'}{(\ell_1, \ell_2) \xrightarrow{g_2:a,D_2} (\ell_1, \ell_2')} \text{ if } a \notin H$$

Example

Semantic w.r.t. Clocks

- Given $g \in CC(C)$ and $\alpha : C \rightarrow \mathbb{R}^{\geqslant 0}$ the meaning of $\alpha \models g$ is obvious
- For $d \in \mathbb{R}^{\geqslant 0}$ and α define $\alpha + d$ as

$$(\alpha + d)(x) = \alpha(x) + d$$

- For $D \subseteq C$ and α define $\alpha[D := 0]$ as

$$\alpha[D := 0](x) = \begin{cases} 0 & \text{if } x \in D \\ \alpha(x) & \text{otherwise} \end{cases}$$

Transition System for a Timed Automaton

For $TA = (Loc, Act, C, \rightarrow, Loc_0, Inv, AP, L)$ obtain transition system:

- States = Locations + Clock-Evaluation (infinite number of states)
- Discrete Transitions: perform (classical) transition
- Delay Transitions: let time pass and stay in a location

Formally: $TS(TA)$ is the transition system $(S, Act', \rightarrow', I, AP, L')$

- $S = Loc \times (C \rightarrow \mathbb{R}^{\geqslant 0})$
- $Act' = Act \cup \mathbb{R}^{\geqslant 0}$
- $I = \{(\ell, \alpha) \mid \ell \in Loc_0, \alpha \models Inv(\ell)\}$ where $\alpha(x) = 0$ for all $x \in C$
- $L'((\ell, \alpha)) = L(\ell)$
- $\rightarrow'$ is composed of two parts: discrete and delay transitions

Transitions of $TS(TA)$

Discrete Transition

$$(\ell,\alpha) \rightarrow_a (\ell',\alpha[D := 0]) \qquad \text{iff}$$

- $\ell \xrightarrow{g:a,D} \ell'$ is transition in TA
- $\alpha \models g$
- $\alpha[D := 0] \models Inv(\ell')$

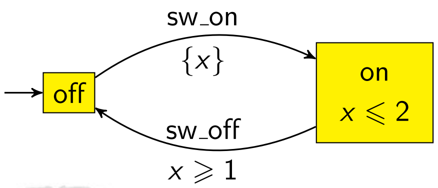
Delay Transition

$$(\ell,\alpha) \rightarrow_d (\ell,\alpha + d) \qquad \text{iff}$$

- $\alpha + d \models Inv(\ell)$ and $d \in \mathbb{R}^{\geqslant 0}$

(This implies that $\alpha + d' \models Inv(\ell)$ for all $0 \leqslant d' \leqslant d$)

Example



Progress of Time

Essentially, the semantics of TA is obtained from paths in $TS(TA)$

For each path $\pi = s_0 \rightarrow_{\tau_0} s_1 \rightarrow_{\tau_1} s_2 \rightarrow_{\tau_2} \dots$ define its execution time as

$$ExecTime(\pi) = \sum_{\tau_i \in \mathbb{R}^{\geqslant 0}} \tau_i$$

For semantics of TA there are certain undesired paths / states in $TS(TA)$

- Time convergent paths (will be ignored)
- States which are timelocks (modeling flaw)
- Zeno paths (modeling flaw)

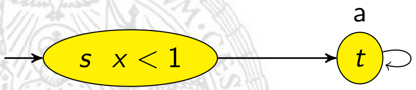
Time Convergent Paths

Race of Achilles (10m/s) versus some fast turtle (1m/s)
Turtle gets 100m in advance

Time elapsed	Achilles	Turtle
0s	0m	100m
10s	100m	110m
11s	110m	111m

Every time Achilles reaches previous point of turtle, the turtle is already a bit ahead. Thus, the turtle wins!?!

Problem: The above claim is only valid for time-points < 11.111...s
Similar problem: time convergent paths π which satisfy $ExecTime(\pi) < \omega$



Does TA satisfy formula A F a? Yes, time-convergent paths like $s \rightarrow_{\frac{1}{2}} s \rightarrow_{\frac{1}{4}} s \rightarrow_{\frac{1}{8}} \dots$ will be ignored. Only consider time divergent paths!

Design Flaws

Usually, time-convergent paths cannot be avoided and will just be ignored for model-checking

The following kinds of phenomena are seen as design flaws and the user has to modify the timed automata to get rid of these phenomena

- A state s is a time-lock if there is no time-divergent path starting in s. TA has a time-lock if there is some reachable state s of $TS(TA)$ which is a time-lock.
  Problem of time-locks: Time cannot proceed beyond certain point
- A path π is zeno if it is time-convergent and contains infinitely many actions $a \in Act$. TA is zeno if there is some (initial) path in $TS(TA)$ which is zeno.
  Problem of zeno paths: infinitely actions in finite time, unrealistic

---

Dealing with Design Flaws

- First step: Apply algorithm to detect time-locks and zeno-paths
- Second step: Fix problem
  Example: one way to avoid zeno paths is to add $x \geqslant$ "small value" as additional guard to actions where additionally it is ensured that x is reseted before

---

Timed Computational Tree Logic (TCTL)

A TCTL-state formula Φ has the following form:

$$\Phi ::= a \mid g \mid \Phi \wedge \Phi \mid \neg\Phi \mid \mathsf{E}\,\psi \mid \mathsf{A}\,\psi$$

where $a \in AP$ is atomic proposition and $g \in CC(C)$ is clock constraint, and ψ is a TCTL-path formula

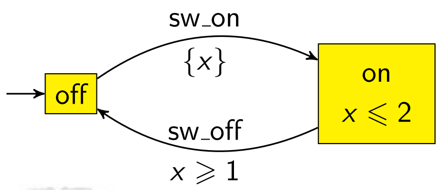
$$\psi ::= \Phi\, \mathsf{U}^{J} \Phi$$

where $J \subseteq \mathbb{R}^{\geqslant 0}$ is an interval with bounds in $\mathbb{N} \cup \{\infty\}$

The connectives $\vee$, true, ... are derived as usual. Moreover,

$$\begin{aligned} \mathsf{F}^{J}\Phi &\equiv \text{true}\, \mathsf{U}^{J}\Phi \\ \mathsf{E}\,\mathsf{G}^{J}\Phi &\equiv \neg \mathsf{A}\,\mathsf{F}^{J}\neg\Phi \\ \mathsf{A}\,\mathsf{G}^{J}\Phi &\equiv \neg \mathsf{E}\,\mathsf{F}^{J}\neg\Phi \end{aligned}$$

Examples



Towards the Semantics of TCTL

- Already stated: only time-divergent paths are considered
- Compare

$$s_1 \longrightarrow_{\frac{1}{6}} s_1 + \frac{1}{6} \longrightarrow_{\frac{7}{6}} s_1 + \frac{4}{3} \longrightarrow_a s_2 \longrightarrow_4 s_2 \longrightarrow_b s_3 \ldots$$

  with

$$s_1 \longrightarrow_{\frac{4}{3}} s_1 + \frac{4}{3} \longrightarrow_a s_2 \longrightarrow_4 s_2 + 4 \longrightarrow_b s_3 \ldots$$

  Both paths are equivalent and we only consider the latter one where consecutive delay-transitions are merged into one delay-transition
- Afterwards merge each delay-transition with the following discrete transition to compressed path. Since actions are ignored by (T)CTL, only denote the consumed time:

$$s_1 \longrightarrow_{\frac{4}{3}} s_2 \longrightarrow_4 s_3 \ldots$$

- If compressed path contains only finitely many discrete transitions then use $\longrightarrow_1$-steps until infinity: $s_1 \longrightarrow_{\frac{4}{3}} s_2 \longrightarrow_1 s_2 + 1 \longrightarrow_1 s_2 + 2 \ldots$

Semantics of TCTL

Let $TA = (Loc, Act, C, \longrightarrow, Loc_0, Inv, AP, L)$. Then a state s of $TS(TA)$ has the form (ℓ, α) where $\ell \in Loc$ and α is a clock-evaluation.

- $s \models a$ iff $a \in L(\ell)$
- $s \models g$ iff $\alpha \models g$
- $s \models \neg\Phi$ iff $s \not\models \Phi$
- $s \models \Phi \wedge \Psi$ iff $s \models \Phi$ and $s \models \Psi$
- $s \models \mathsf{E}\,\varphi$ iff there is some time-divergent compressed path π: $\pi \models \varphi$
- $s \models \mathsf{A}\,\varphi$ iff for all time-divergent compressed paths π: $\pi \models \varphi$

Semantics of TCTL (continued)

Let π be a time-divergent compressed path

$$\pi = s_0 \longrightarrow_{d_0} s_1 \longrightarrow_{d_1} s_2 \longrightarrow_{d_2} s_3 \ldots$$

Then $\pi \models \Phi \,\mathsf{U}^J \Psi$ iff

- there is some i such that $s_i + d \models \Psi$ for some $d \in [0, d_i]$ with

$$d + \sum_{k=0}^{i-1} d_k \in J$$

  and for all $j \leqslant i$ and all $d' \in [0, d_j]$ such that

$$d' + \sum_{k=0}^{j-1} d_k \leqslant d + \sum_{k=0}^{i-1} d_k$$

  the relation $s_j + d' \models \Phi \vee \Psi$ is valid

As usual, $TA \models \Phi$ iff all initial states satisfy Φ

Some Notes on TCTL

- There is no next-operator (X) since it is unclear what the next point in time should be
- The intervals need not be hit by state of path, e.g.,

$$s_0 \longrightarrow_1 s_1 \longrightarrow_4 s_2 \ldots \models \mathsf{F}^{[2,3]} a$$

  provided that $a \in L(s_1)$
- The semantics of until requires that the left formula is satisfied from now on until the right- formula is satisfied, and not only from the start of J onwards, e.g.,

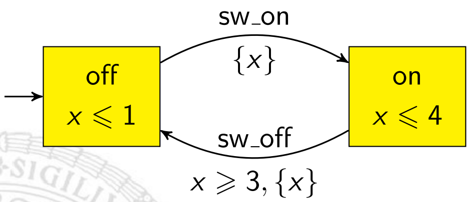
$$s_0 \longrightarrow_1 s_1 \longrightarrow_4 s_2 \cdots \not\models a \,\mathsf{U}^{[2,3]} a$$

  provided that $a \notin L(s_0)$, $a \in L(s_1)$

More Notes on TCTL

In CTL: $\pi \models \Phi \,\mathsf{U}\, \Psi$ … and for all $j \leqslant i : s_j \models \Phi$
In TCTL: $\pi \models \Phi \,\mathsf{U}^J \Psi$ … and for all $j \leqslant i : s_j + d' \models \Phi \vee \Psi$

- Not a real difference in CTL since $\Phi \,\mathsf{U}\, \Psi \equiv_{CTL} (\Phi \vee \Psi) \,\mathsf{U}\, \Psi$
- Allows early satisfaction of right formula:



$$TA \models \mathsf{A}\, \mathsf{off}\, \mathsf{U}^{[1,2]} \mathsf{on}$$

Example

Overview

Main question:

$$TA \models \Phi$$

for timed automata TA and TCTL-state-formula Φ

- Know: $TA \models \Phi$ iff $TS(TA) \models \Phi$
- First Problem: $TS(TA)$ has infinitely many states
- Solution: Construct region transition system RTS (quotient of $TS(TA)$) with finitely many states

- Second Problem: How to deal with intervals J in $\Psi_1 \,\mathsf{U}^J \Psi_2$
- Solution: Add additional clock which allows to transform Φ into CTL-formula Ψ

⇒ $TA \models \Phi$ iff $RTS \models \Psi$

⇒ TCTL-model checking boils down to CTL-model checking

- Restriction: From now only consider non-zeno timed automata

---

Idea of Region Transition System

Goal: Checking $TA \models \Phi$. Let x be some clock of TA

- First observation: All clock constraints consists of atoms $x < / \leqslant / \geqslant / > c$ for some $c \in \mathbb{N}$

⇒ It does not matter whether $x = 2.334$ or $x = 2.893$, both values of x satisfy the same clock constraints

⇒ Abstract from concrete value of x, only consider following intervals:

$$\{0\}, (0,1), \{1\}, (1,2), \{2\}, (2,3), \ldots$$

⇒ Far less values, but still infinitely many

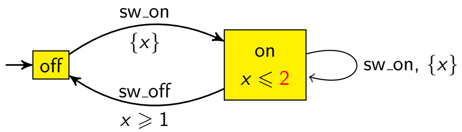
- Second observation: There is some largest constant c_x which occurs in a clock constraints about x in TA and Φ

⇒ The following finite set of intervals suffices:

$$\{0\}, (0,1), \{1\}, (1,2), \ldots, (c_x - 1, c_x), \{c_x\}(c_x, \infty)$$

By just looking at these intervals one can still decide all clock constraints which occur in TA and Φ

Example



Clocks: $C = \{x, y\}$. Question: $TA \models \mathsf{A}\,\mathsf{F}\,(y = 3 \wedge \text{off})$?

- States in $TS(TA)$: (ℓ, x, y) with $\ell \in \{\text{on, off}\}$, $x, y \in \mathbb{R}^{\geqslant 0}$
- From TA and Φ extract $c_x = 2$ and $c_y = 3$
- States in region transition system RTS: (ℓ, x, y) with $\ell \in \{\text{on, off}\}$ and (x, y) is one of the following 48 regions (point, line segment, or white area):

Delay Transitions in Region Transition System

- Have: finite region transition system, fine enough to decide clock constraints
- ⇒ Possible to mimic discrete transitions within region transition system
  - Guard of transition can be checked by region
  - Resetting clocks can be done directly with regions
  - Invariant of locations can be checked by region

- Region transition system is not fine enough to mimic delay transitions:
  - Consider clocks x, y and region $R =$ "$x \in (0,1) \wedge y \in (2,3)$"
  - Want to compute the next region. Candidates:

$$x = 1 \wedge y \in (2,3) \quad \text{or} \quad x = 1 \wedge y = 3 \quad \text{or} \quad x \in (0,1) \wedge y = 3$$

  - Problem: all three cases are possible when starting in R

$$x = 0.8, y = 2.3 \quad \text{or} \quad x = 0.7, y = 2.7 \quad \text{or} \quad x = 0.4, y = 2.9$$

  - Solution: construct finer regions where additionally the fractional parts of clock values are compared with $\leqslant$

Example

With refinement obtain 12 additional regions

Regions (Formally)

$frac(d)$ denotes the fractional part of d, $\lfloor d \rfloor$ denotes the integral part of d

Definition (Clock Equivalence, Region, Unbounded Region)

Let $\alpha, \beta \in C \to \mathbb{R}^{\geqslant 0}$ be clock valuations. Let $c_x, c_y, \ldots$ be the maximal occurring constants. Then α and β are clock equivalent ($\alpha \cong \beta$) iff one of the following two conditions are satisfied

- for all $x \in C$: $\alpha(x) > c_x$ and $\beta(x) > c_x$
- for all $x, y \in C$ where $\alpha(x), \beta(x) \leqslant c_x$ and $\alpha(y), \beta(y) \leqslant c_y$ the following two conditions are satisfied:
  - $\lfloor \alpha(x) \rfloor = \lfloor \beta(x) \rfloor$ and $frac(\alpha(x)) = 0$ iff $frac(\beta(x)) = 0$
  - $frac(\alpha(x)) \leqslant frac(\alpha(y))$ iff $frac(\beta(x)) \leqslant frac(\beta(y))$

The regions are the equivalence classes of $\cong$

The unbounded region R_∞ is the equivalence class of α (i.e, $R_\infty = [\alpha]_{\cong}$) where $\alpha(x) = c_x + 1$, for all $x \in C$

Number of Regions

Let α be a clock valuation. The corresponding region is identified by

- integral parts α, i.e., by $\lfloor\alpha(x)\rfloor, \lfloor\alpha(y)\rfloor, \dots$

$$\prod_{x \in C} c_x + 1 \text{ possibilities}$$

- being an natural number or not, i.e., by bits $\mathit{frac}(\alpha(x)) = 0, \dots$

$$2^{|C|} \text{ possibilities}$$

- order of fractional parts, e.g., $\mathit{frac}(\alpha(x)) = \mathit{frac}(\alpha(z)) < \mathit{frac}(\alpha(y))$

$$|C|! \cdot 2^{|C|-1} \text{ possibilities}$$

⇒ number of regions is bounded by

$$(\prod_{x \in C} c_x + 1) \cdot 4^{|C|} \cdot |C|!$$

⇒ size of region transition system is exponential in number of clocks

---

Successor of a Region

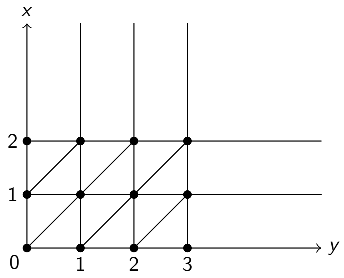
For each region R there is a unique successor region $\mathit{succ}(R)$:

- If $R = R_\infty$ then $\mathit{succ}(R) = R_\infty$
- If $R \neq R_\infty$ then $\mathit{succ}(R)$ is the unique region R' such that $R' \neq R$ and for all $\alpha \in R$:

$$\exists d > 0 : (\alpha + d \in R' \text{ and } \forall 0 \leqslant d' \leqslant d : \alpha + d' \in R \cup R')$$

So, $R' \neq R$ is the region that is visited next when starting in R

Example



$x = 0 \wedge 1 < y < 2$ $\rightarrow_{succ}$ $0 < x < 1 \wedge 1 < y < 2 \wedge \mathit{frac}(x) < \mathit{frac}(y)$
$\rightarrow_{succ}$ $0 < x < 1 \wedge y = 2$
$\rightarrow_{succ}$ $0 < x < 1 \wedge 2 < y < 3 \wedge \mathit{frac}(x) > \mathit{frac}(y)$
$\rightarrow_{succ}$ $x = 1 \wedge 2 < y < 3$
$\rightarrow_{succ}$ $1 < x < 2 \wedge 2 < y < 3 \wedge \mathit{frac}(x) < \mathit{frac}(y)$
$\rightarrow_{succ}$ $1 < x < 2 \wedge y = 3$
$\rightarrow_{succ}$ $1 < x < 2 \wedge y > 3$
$\rightarrow_{succ}$ $x = 2 \wedge y > 3$
$\rightarrow_{succ}$ $R_\infty : x > 2 \wedge y > 3$

Region Transition System

Definition

Let $TA = (Loc, Act, C, \rightarrow, Loc_0, Inv, AP, L)$ and TCTL-formula Φ be given. Then $RTS(TA, \Phi)$ is the region transition system

$$RTS = (Loc \times (C \rightarrow \mathbb{R}^{\geqslant 0} / \approxeq), \rightarrow', I, AP \cup CC(\Phi), L')$$

- $C \rightarrow \mathbb{R}^{\geqslant 0} / \approxeq$ are the clock evaluations modulo $\approxeq$, i.e, the regions
- $I = \{(\ell, [\alpha]_\approxeq) \mid \ell \in Loc_0, \alpha \models Inv(\ell)\}$ where $\alpha(x) = 0$ for all $x \in C$
- $CC(\Phi)$ are the clock-constraints that are occurring in Φ
- $L'((\ell, R)) = L(\ell) \cup \{g \in CC(\Phi) \mid R \models g\}$
- $(\ell, R) \rightarrow' (\ell, R')$ if $\mathit{succ}(R) = R'$ and $R' \models Inv(\ell)$
- $(\ell, R) \rightarrow' (\ell', R[D := 0])$ if
  - $\ell \xrightarrow{g:a,D} \ell'$ is transition in TA
  - $R \models g$
  - $R[D := 0] \models Inv(\ell')$

Remarks on Region Transition System

- $R[D := 0] = \{\alpha[D := 0] \mid \alpha \in R\}$
- $R \models g$ iff for all $\alpha \in R : \alpha \models g$ iff there exists $\alpha \in R : \alpha \models g$

$\Rightarrow$ There is no ambiguity in the labeling

- $\approx$ needs values $c_x, c_y, \dots$. These are extracted from TA and Φ
- Clock constraints of Φ seen as TCTL-formula become atomic propositions in $RTS(TA, \Phi)$

Example

Properties of Region Transition System

Recall: only timed automata are considered which are non-zeno

Theorem

TA has a time-lock iff $RTS(TA, true)$ has a reachable terminal state

$\Rightarrow$ Directly yields method to check for time-locks

Theorem

$$TA \models \Phi \text{ iff } RTS(TA, \Phi) \models \Phi$$

($TS(TA)$ is bisimilar to $RTS(TA, \Phi)$ w.r.t. AP' where AP' does not contain guards exceeding $c_x, c_y, \dots$)

$\Rightarrow$ Perform CTL-model checking on finite system to answer $TA \models \Phi$

One Remaining Problem

Now: standard CTL-model checking on RTS applicable to answer $TA \models \Phi$

With this approach cover

$$\Phi ::= a \mid g \mid \Phi \wedge \Phi \mid \neg\Phi \mid \mathsf{E}\,\psi \mid \mathsf{A}\,\psi$$

where

$$\psi ::= \Phi \,\mathsf{U}\, \Phi$$

However, unclear how to handle $\Phi \,\mathsf{U}^{J} \Phi$ where $J \neq [0, \infty)$

Elimination of Timing Parameters

Aim: Get rid of J in $\Phi \, \mathrm{U}^J \Psi$

Solution:

- Add fresh clock z to TA, obtain $TA \uplus \{z\}$
  (z is not reseted, neither contained in guards nor in invariants)

$\Rightarrow$ z counts global elapsed time

- Lift $Sat(\Phi)$ and $Sat(\Psi)$ from TA to $TA \uplus \{z\}$
- Replace $\Phi \, \mathrm{U}^J \Psi$ by $\xi := (\Phi \vee \Psi) \, \mathrm{U} \, (z \in J \wedge \Psi)$

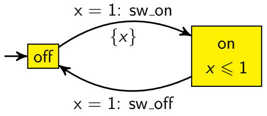
Theorem (Elimination of Timing Parameters)

- $s \models \mathrm{E} \, \Phi \, \mathrm{U}^J \Psi$ *iff* $s[\{z\} := 0] \models \mathrm{E} \, \xi$ *(pure CTL model-checking)*
- $s \models \mathrm{A} \, \Phi \, \mathrm{U}^J \Psi$ *iff* $s[\{z\} := 0] \models \mathrm{A} \, \xi$ *(pure CTL model-checking)*

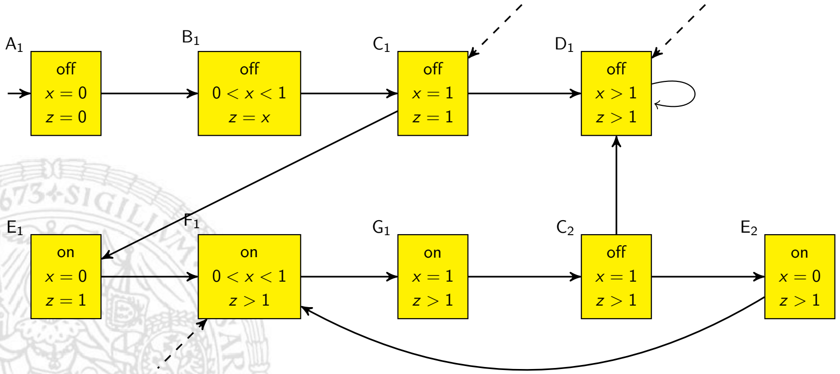
Here, s is state of $RTS(TA, \dots)$ and $s[\{z\} := 0]$ is state of $RTS(TA \uplus \{z\}, \dots)$. Note that for building $RTS(TA \uplus \{z\}, \dots)$ one also has to consider the clock constraint $z \in J$ which determines c_z

---

Example



parts of $RTS(TA \uplus \{z\}, \dots z < 1 \dots)$



Example

Summary

- Often modeling is only adequate if real-time aspects can be expressed
- Complex real-time systems can be modeled via composition of timed automata (containing clocks, guards, invariants)
- Timed CTL is extension of CTL where until-operator is equipped with intervals
- Model-checking for timed CTL possible via region transition system (but exponential in number of clocks)