Model Checking (VO) SS 2008 LVA 703521

First name: ____________________

Last name: ____________________

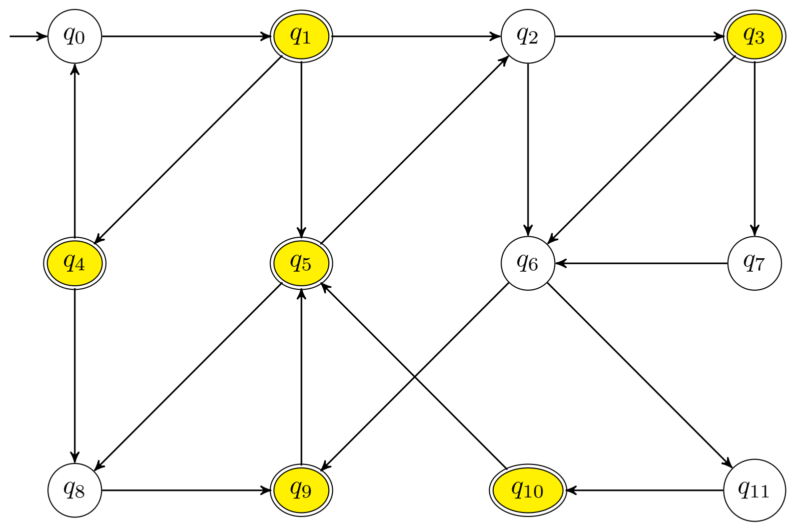
Matriculation number: ____________________

- Write your name and matriculation number on every page.
- Please answer all exercises in a readable and precise way. Please cross out solution attempts which are replaced by another solution.
- Cheating is not allowed. Everyone who is caught will fail the exam.
- Please do not remove the staples of the exam.

Exercise	Maximal points	Points
1	22	
2	24	
3	35	
4	19	
Σ	100	
Grade		

Exercise 1 (18 + 4 points)

Consider the following NBA $\mathcal{A}$ (where labels are omitted).



- Fill the table which is obtained when using the algorithm of Yannakakis et. al. where successors are taken in order, i.e., successor q_i is taken before q_j iff $i < j$.

  Here, you should at least write down a new line whenever the set of marked or flagged states are changed. You may abbreviate q_i by i and you may use $\{1-4, 6\}$ to indicate the set $\{q_1, q_2, q_3, q_4, q_6\}$, etc.

outer_dfs-stack	**inner_dfs-stack**	**marked**	**flagged**
ε	ε	$\varnothing$	$\varnothing$
q_0	ε	$\{q_0\}$	$\varnothing$
$q_1,\ q_0$	ε	$\{q_0,\ q_1\}$	$\varnothing$
$q_2,\ q_1,\ q_0$	ε	$\{q_0,\ q_1,\ q_2\}$	$\varnothing$
$q_3,\ q_2,\ q_1,\ q_0$	ε	$\{q_0,\ q_1,\ q_2,\ q_3\}$	$\varnothing$
$q_6,\ q_3,\ q_2,\ q_1,\ q_0$	ε	$\{q_0,\ q_1,\ q_2,\ q_3,\ q_6\}$	$\varnothing$
$q_9,\ q_6,\ q_3,\ q_2,\ q_1,\ q_0$	ε	$\{q_0,\ q_1,\ q_2,\ q_3,\ q_6,\ q_9\}$	$\varnothing$
$q_5,\ q_9,\ q_6,\ q_3,\ q_2,\ q_1,\ q_0$	ε	$\{q_0,\ q_1,\ q_2,\ q_3,\ q_5,\ q_6,\ q_9\}$	$\varnothing$
$q_8,\ q_5,\ q_9,\ q_6,\ q_3,\ q_2,\ q_1,\ q_0$	ε	$\{q_0,\ q_1,\ q_2,\ q_3,\ q_5,\ q_6,\ q_8,\ q_9\}$	$\varnothing$
$q_5,\ q_9,\ q_6,\ q_3,\ q_2,\ q_1,\ q_0$	q_5	$\{q_0,\ q_1,\ q_2,\ q_3,\ q_5,\ q_6,\ q_8,\ q_9\}$	$\{q_5\}$

- Is there an accepting run of $\mathcal{A}$? If so, which run is extracted by the algorithm?

  The algorithm extracts the accepting run $q_0\ q_1\ (q_2\ q_3\ q_6\ q_9\ q_5)^\omega$.

Exercise 2 (12 + 12 points)

- Consider the following language:

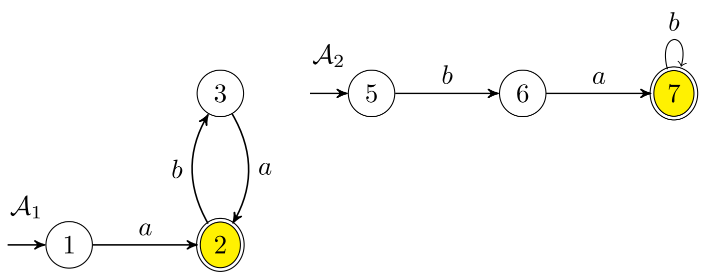
$$\begin{pmatrix}0\\1\end{pmatrix}^* \begin{pmatrix}0\\0\end{pmatrix} \left(\begin{pmatrix}1\\1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix}\right)^\omega$$

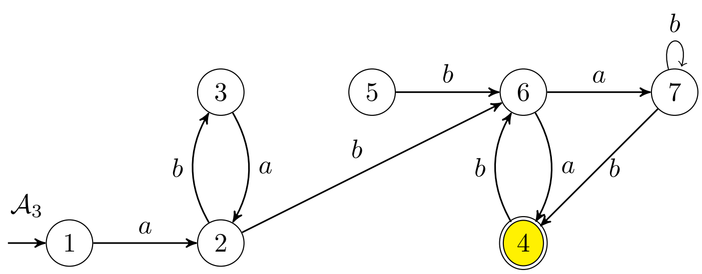
  Write down an F1S- or S1S-formula for this language and shortly explain your formula. If you used S1S, do you think it is possible with F1S?

  x corresponds to the point in time where $\begin{pmatrix}0\\0\end{pmatrix}$ occurs.

$$\exists x: \quad \neg \mathsf{a}_2(x) \wedge \mathsf{a}_2(x') \wedge \neg \mathsf{a}_2(x'') \wedge \forall y:$$
$$(x < y \Leftrightarrow \mathsf{a}_1(y)) \wedge (y < x \Rightarrow \mathsf{a}_2(y)) \wedge ((x < y \wedge \neg \mathsf{a}_2(y)) \Rightarrow (\mathsf{a}_2(y') \wedge \mathsf{a}_2(y'') \wedge \neg \mathsf{a}_2(y''')))$$

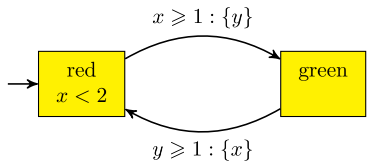
- Intuitively construct NFAs $\mathcal{A}_1$ and $\mathcal{A}_2$ for the languages $a(ba)^*$ and bab^*. Then construct an NBA $\mathcal{A}_3$ for the language $(a(ba)^*) \cdot (bab^*)^\omega$ using the construction from the lecture. Write down all three automata $\mathcal{A}_1$, $\mathcal{A}_2$, and $\mathcal{A}_3$.



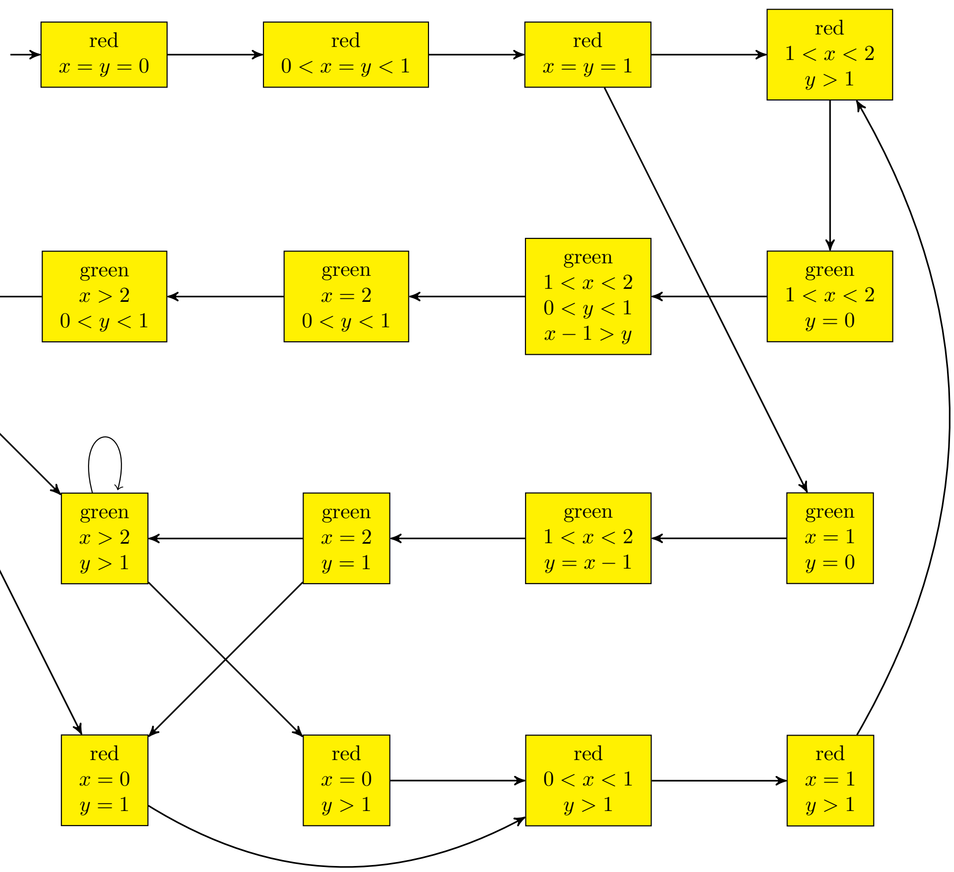


Exercise 3 (34 + 1 points)

Consider the following timed automaton.



- Construct the reachable part of the region transition system for this automaton. (It has 17 states.)



- Does the timed automata have a time-lock?

  Since there are no reachable terminal states in the region transition system, the timed automata does not have a time-lock.

Exercise 4 (19 points)

Recall the definition of simulation equivalence $\simeq$:

- $TS_1 = (S_1, \to_1, I_1, AP, L_1) \preceq TS_2 = (S_2, \to_2, I_2, AP, L_2)$ if there is a simulation relation R for TS_1 and TS_2:
  - for all $s \in I_1$ there exists $t \in I_2$: sRt
  - whenever sRt then $L_1(s) = L_2(t)$
  - whenever sRt and $s \to_1 s'$ then there exists t': $t \to_2 t'$ and $s'Rt'$
- $\simeq \, = \, \preceq \cap \succeq$

Proof that $\simeq$ is an equivalence relation (reflexive, symmetric, transitive). Whenever you define some simulation relation R then you do not have to formally show that R really is a simulation relation.

- $TS_1 \simeq TS_1$, i.e., $\simeq$ is reflexive: the reason is that $R = \{(s, s) \mid s \in S_1$ is simulation relation for TS_1 and TS_1 proving both $TS_1 \preceq TS_1$ and $TS_1 \succeq TS_1$, and hence $TS_1 \simeq TS_1$.
- Let $TS_1 \simeq TS_2$. Then $TS_1 \preceq TS_2$ and $TS_1 \succeq TS_2$ and thus, $TS_2 \simeq TS_1$. Thus, $\simeq$ is symmetric.
- We first show that $\preceq$ is transitive. So, let $TS_1 \preceq TS_2 \preceq TS_3$. Then there are simulations R_{12} for TS_1 and TS_2, and R_{23} for TS_2 and TS_3. Then $R = \{(s_1, s_3) \mid \exists s_2 : s_1 R_{12} s_2 \text{ and } s_2 R_{23} s_3\}$ is a simulation for TS_1 and TS_3. Thus, $TS_1 \preceq TS_3$ which proves transitivity of $\preceq$.

  Now let $TS_1 \preceq \cap \succeq TS_2 \preceq \cap \succeq TS_3$. This implies $TS_1 \preceq TS_2 \preceq TS_3$ and $TS_1 \succeq TS_2 \succeq T_3$ which by transitivity of $\preceq$ implies $TS_1 \preceq TS_3$ and $TS_1 \succeq TS_3$, i.e., $TS_1 \simeq TS_3$. Hence, $\simeq$ is transitive.