Algorithms & Datastructures
Laboratory Exercise Sheet 6

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Exercise 1) Tree Traversal The tree iterator from the lecture only supports traversal of trees, but did not allow to modify the underlying data-structure. For this exercise you should also implement the `remove()`-method in the iterator with the following conventions (adapted from the Java Iterator-Interface).

- `remove` may only be invoked after a call to `next`.
- If `remove` is called, then the last element that has been returned by `next` is removed from the tree. It is not allowed to call `remove` twice without an intermediate call to `next`.
- If `remove` is called wrongly, an `IllegalStateException` should be thrown.
- After a (legal) call to `remove`, the iterator should proceed normally, i.e., in total, all elements must be traversed and no element should be returned twice.

For example, if there is a tree with elements 4,7,8,10,11 then after calling `next` twice, the iterator may return first the 4 and then the 8 (or other values, depending on the structure of the tree and the traversal-order). A call of `remove` should now delete the 8 from the tree. Two more calls to `next` return the values 11 and 7. Another call to `remove` is possible and removes the 7. Finally, one more call to `next` is possible and it returns the element 10.

1. In the sources to this exercise sheet you already find a slightly simplified binary tree implementation from the lecture where the iterator only supports inorder-traversal. Modify this implementation such that the iterator supports the `remove`-operation. (You might also need to change other methods, please indicate which methods have been changed.)
2. Would your implementation also work correct for preorder-traversal or for postorder-traversal? If not, try to illustrate the problem (think about removal of nodes with two children).

Exercise 1) Solution

1. Essentially, we store the node that was previously returned by `next`. If there is such a node we remove it by the `removeNode`-method from the `BinTree`-class and set the previously returned node to `null`. Otherwise, `remove` was called wrongly and we throw an exception.

   However, to be able to call the `removeNode`-method, we have to also store the tree in the iterator. Therefore, the signature of the constructor of the `TreeIterator` has to be changed, too, and likewise the invocation of this constructor within the `BinTree`-class.

   In total, we have the following changes (in lines 4,5,7,8,9,11,19,26-31,37):

```
1  public class TreeIterator<D> implements Iterator<D> {
2     int direction;   // from which direction ...
3     Node<D> current; // ... did we enter current node
4     Node<D> previous; // the node we previously returned by next()
5     BinTree<D> tree; // the tree we are traversing
6
7     public TreeIterator(BinTree<D> tree) {
8        this.current = tree.root;
9        this.tree = tree;
10       this.direction = UP;
11       this.previous = null;
12       walkNext(INORDER);
13    }
14    public Entry<D> next() {
15       if (current == null) {
16          throw new NoSuchElementException();
17       } else {
18          Entry<D> entry = new Entry<D>(current.key, current.data);
19          this.previous = this.current;
20          walkNext(NONE);
21          return entry;
22       }
23    }
24
25    public void remove() {
26       if (this.previous == null) {
27          throw new IllegalStateException();
28       } else {
29          this.tree.removeNode(this.previous);
30          this.previous = null;
31       }
32    }
33
34 }
```

```
35 public class BinTree<D> implements Dictionary<D> {
36    public Iterator<D> iterator() {
37       return new TreeIterator<D>(this);
38    }
39
40 }
```

2. The iterator does not work correctly for the preorder traversal. Consider a tree that is obtained after inserting values 3,1,2,4 in that order. Then an initial call to `next` will return the 3 and the `current`-reference is at the node with the 1. Now calling `remove` will remove the node with the 3 which has two children, so the 3 and the 2 are swapped and then the 3 is deleted (where now the 2 is the root of the node). As the iterator has already output

the root-node, the following calls to `next` will result in 1 and 4, so the 2 is never returned by the iterator.

The iterator will however work correctly for the postorder traversal: whenever a value in a node is returned then the iterator will not revisit and output nodes in the corresponding subtree. Hence, it is safe to remove such a node. (A similar reasoning also shows that there are no problems using the inorder-traversal.)

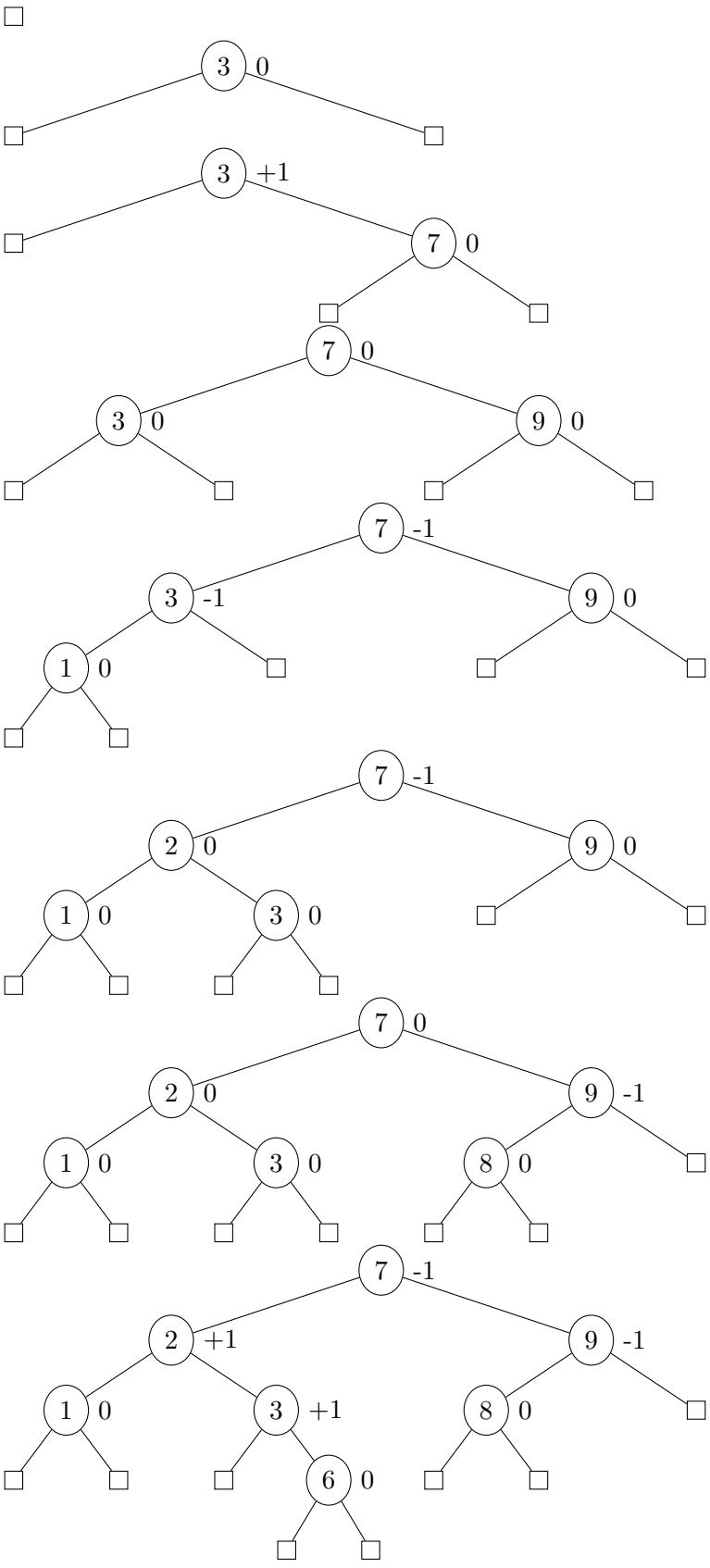
Exercise 2) AVL Trees

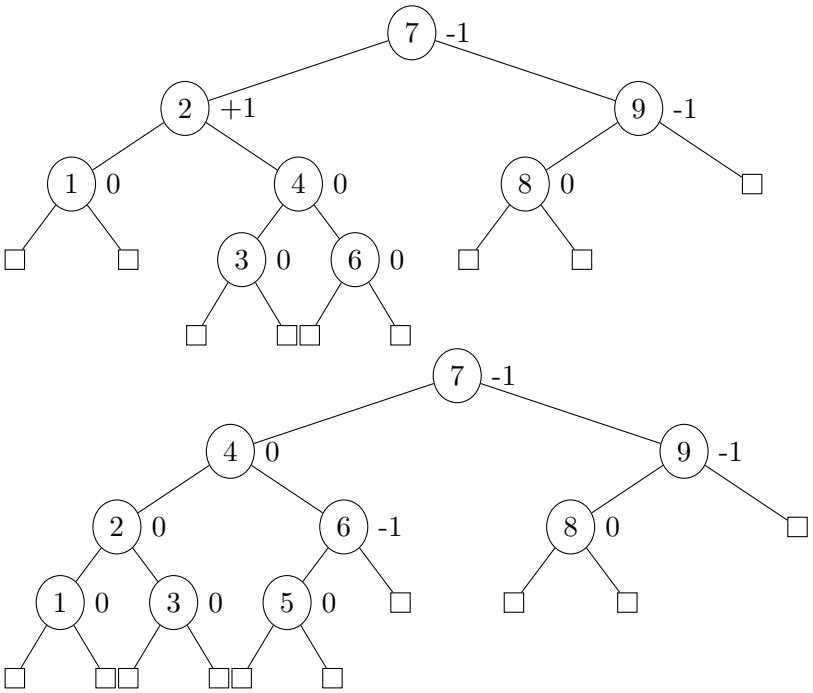
1. Insert the values 3,7,9,1,2,8,6,4,5 into an empty AVL-tree (in the given order). Give the tree after each insertion.
2. In the picture on slide 163 the new `hdiff`-values for nodes `n` and `p` are missing. Determine these values by only looking at the other `hdiff`-values, but without computing the heights of the trees.
3. Implement the method `rotateLeft` for AVL-tree nodes. This method should update all references to children, parents, etc., but should ignore the `hdiff`-values.
4. Implement `balGrow` where you can omit the symmetric cases where `n = p.right`. Moreover, you can assume that both `rotateLeft` and `rotateRight` are available.

An incomplete implementation of AVL-trees is given in the sources of this exercise sheet.

Exercise 2) Solution

1.





2.-4.

```java
public class AVLTree<D> implements Dictionary<D> {
   AVLNode<D> root;
   /**
    * does not update the balance-informations
    */
   void rotateRight(AVLNode<D> p) {
      AVLNode<D> l = p.left;
      l.parent = p.parent;
      p.left = l.right;
      l.right = p;
      p.parent = l;
      if (p.left != null) {
         p.left.parent = p;
      }
      if (l.parent == null) {
         this.root = l;
      } else if (l.parent.left == p) {
         l.parent.left = l;
      } else {
         l.parent.right = l;
      }
   }

   /**
    * does not update the balance-informations
    */
   void rotateLeft(AVLNode<D> p) {
      AVLNode<D> r = p.right;
      r.parent = p.parent;
      p.right = r.left;
      r.left = p;
      p.parent = r;
      if (p.right != null) {
         p.right.parent = p;
      }
      if (r.parent == null) {
         this.root = r;
```

```
38          } else if (r.parent.left == p) {
39              r.parent.left = r;
40          } else {
41              r.parent.right = r;
42          }
43      }
44
45      void balGrow(AVLNode<D> n) {
46          while (n.parent != null) {
47              AVLNode<D> p = n.parent;
48              if (p.left == n) {
49                  if (p.hdiff == 1) {
50                      p.hdiff = 0;
51                      return;
52                  } else if (p.hdiff == 0) {
53                      p.hdiff = -1;
54                      n = p;
55                  } else if (n.hdiff == -1) { // and p.hdiff = -1
56                      rotateRight(p);
57                      p.hdiff = 0;
58                      n.hdiff = 0;
59                      return;
60                  } else { // n.hdiff = 1, p.hdiff = -1
61                      AVLNode<D> r = n.right;
62                      rotateLeft(n);
63                      rotateRight(p);
64                      n.hdiff = r.hdiff ==  1 ? -1 : 0;
65                      p.hdiff = r.hdiff == -1 ?  1 : 0;
66                      r.hdiff = 0;
67                      return;
68                  }
69              } else { // p.right = n
70                  if (p.hdiff == -1) {
71                      p.hdiff = 0;
72                      return;
73                  } else if (p.hdiff == 0) {
74                      p.hdiff = 1;
75                      n = p;
76                  } else if (n.hdiff == 1) { // and p.hdiff = 1
77                      rotateLeft(p);
78                      p.hdiff = 0;
79                      n.hdiff = 0;
80                      return;
81                  } else { // n.hdiff = -1, p.hdiff = 1
82                      AVLNode<D> l = n.left;
83                      rotateRight(n);
84                      rotateLeft(p);
85                      n.hdiff = l.hdiff == -1 ?  1 : 0;
86                      p.hdiff = l.hdiff ==  1 ? -1 : 0;
87                      l.hdiff = 0;
88                      return;
89                  }
90              }
91          }
92      }
93  }
```