University of Innsbruck
1st exam

Institute of Computer Science
5 June 2008

Model Checking (VO) SS 2008 LVA 703521

First name: ____________________

Last name: ____________________

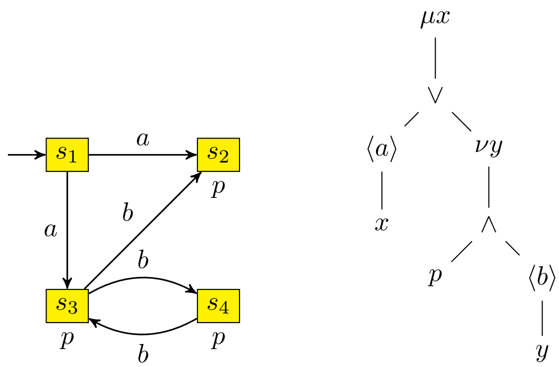
Matriculation number: ____________________

- Write your name and matriculation number on every page.
- Please answer all exercises in a readable and precise way. Please cross out solution attempts which are replaced by another solution.
- Cheating is not allowed. Everyone who is caught will fail the exam.
- Please do not remove the staples of the exam.

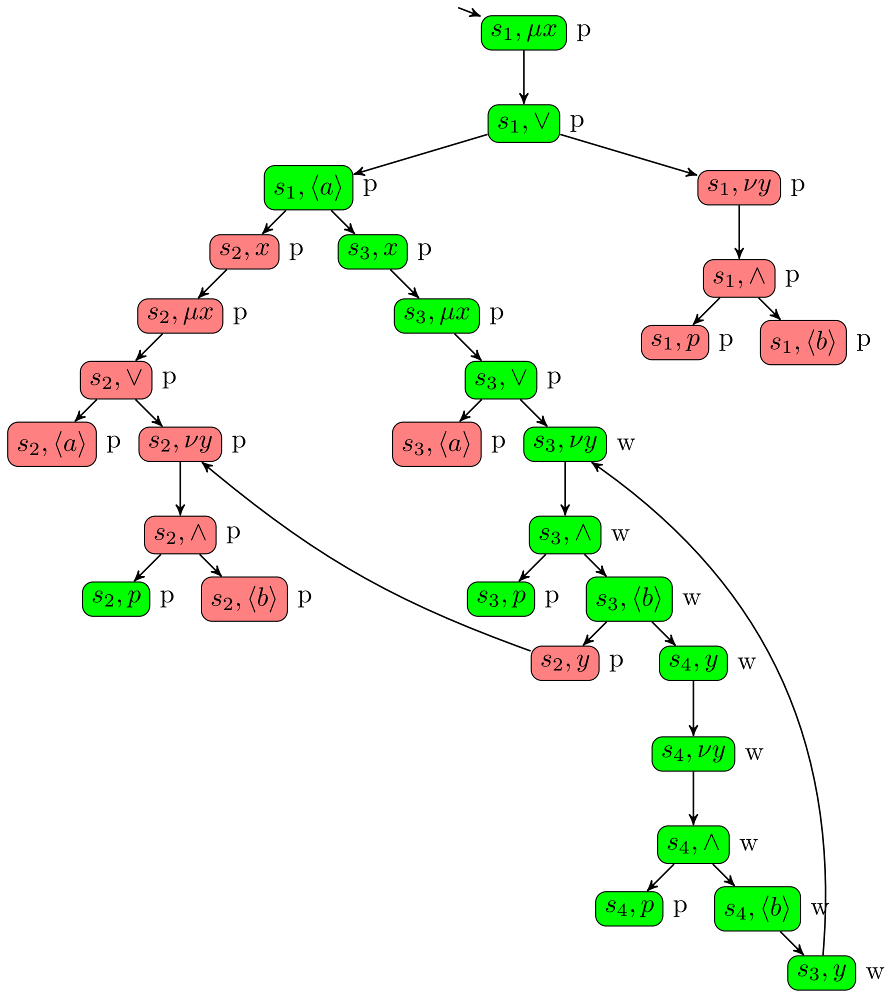
Exercise	**Maximal points**	**Points**
1	**35**	
2	**30**	
3	**20**	
4	**15**	
Σ	**100**	
Grade		

Exercise 1 (20 + 15 points)

Consider the following transition system and formula.

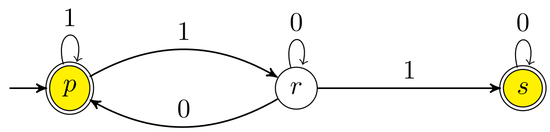


- Complete the game-graph which is depicted on the next page.
- Color the game-graph using the bottom-up coloring algorithm. Mark your nodes by (**g**)reen or (**r**)ed and additionally indicate whether a node was colored during a (**p**)ropagation-phase or whether it was colored since it remained (**w**)hite after propagation stopped. So, label each node with one of {**gp**,**rp**,**gw**,**rw**}.


$s_1, \mu x$ p
$s_1, \vee$ p
$s_1, \langle a \rangle$ p
$s_1, \nu y$ p
s_2, x p
s_3, x p
$s_1, \wedge$ p
$s_2, \mu x$ p
$s_3, \mu x$ p
s_1, p p
$s_1, \langle b \rangle$ p
$s_2, \vee$ p
$s_3, \vee$ p
$s_2, \langle a \rangle$ p
$s_2, \nu y$ p
$s_3, \langle a \rangle$ p
$s_3, \nu y$ w
$s_2, \wedge$ p
$s_3, \wedge$ w
s_2, p p
$s_2, \langle b \rangle$ p
s_3, p p
$s_3, \langle b \rangle$ w
s_2, y p
s_4, y w
$s_4, \nu y$ w
$s_4, \wedge$ w
s_4, p p
$s_4, \langle b \rangle$ w
s_3, y w

Exercise 2 (18 + 12 points)

Consider the following NBA $\mathcal{A}$.



- Compute the $\mathcal{A}$-equivalence classes by giving their shortest representatives and the corresponding transition profiles.

representative w	$tp(w)$
ϵ	$p \twoheadrightarrow_F p, r \twoheadrightarrow r, s \twoheadrightarrow_F s$
0	$r \twoheadrightarrow_F p, r \twoheadrightarrow r, s \twoheadrightarrow_F s$
1	$p \twoheadrightarrow_F p, p \twoheadrightarrow_F r, r \twoheadrightarrow_F s$
0 0	same as 0
0 1	$r \twoheadrightarrow_F p, r \twoheadrightarrow_F r, r \twoheadrightarrow_F s$
1 0	same as 1
1 1	$p \twoheadrightarrow_F p, p \twoheadrightarrow_F r, p \twoheadrightarrow_F s$
0 1 0	same as 0 1
0 1 1	same as 0 1
1 1 0	same as 1 1
1 1 1	same as 1 1

- Construct the S1S-formula $\varphi_{\mathcal{A}}(\mathsf{a})$ using the construction from the lecture (and not the optimized version from the exercises).

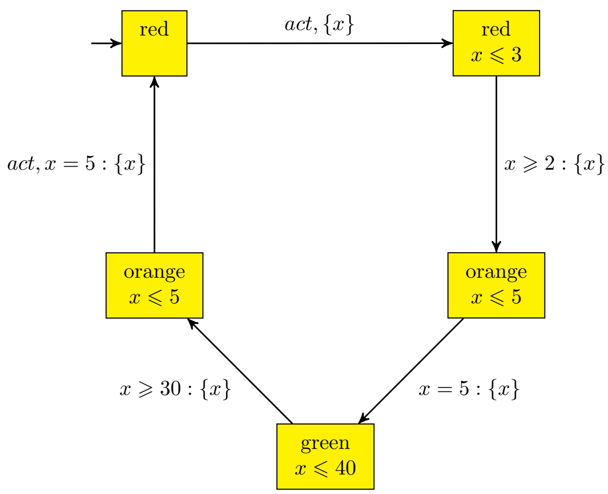
$$
\begin{array}{lll}
\varphi_{\mathcal{A}} = & \exists \mathsf{p} : \exists \mathsf{r} : \exists \mathsf{s} & \\
& \forall x : (\mathsf{p}(x) \vee \mathsf{r}(x) \vee \mathsf{s}(x)) \wedge \neg\left((\mathsf{p}(x) \wedge \mathsf{r}(x)) \vee (\mathsf{p}(x) \wedge \mathsf{s}(x)) \vee (\mathsf{r}(x) \wedge \mathsf{s}(x))\right) \quad \wedge & \text{partition} \\
& \forall x : \exists y : x < y \wedge (\mathsf{p}(y) \vee \mathsf{s}(y)) \quad \wedge & \text{accepting} \\
& \mathsf{p}(0) \wedge \forall x : \quad (\mathsf{p}(x) \wedge \mathsf{a}(x) \wedge \mathsf{p}(x')) \vee & \\
& \qquad (\mathsf{p}(x) \wedge \mathsf{a}(x) \wedge \mathsf{r}(x')) \vee & \\
& \qquad (\mathsf{r}(x) \wedge \neg\mathsf{a}(x) \wedge \mathsf{p}(x')) \vee & \\
& \qquad (\mathsf{r}(x) \wedge \neg\mathsf{a}(x) \wedge \mathsf{r}(x')) \vee & \text{run} \\
& \qquad (\mathsf{r}(x) \wedge \mathsf{a}(x) \wedge \mathsf{s}(x')) \vee & \\
& \qquad (\mathsf{s}(x) \wedge \neg\mathsf{a}(x) \wedge \mathsf{s}(x')) &
\end{array}
$$

Exercise 3 (20 points)

Consider a crossing with two traffic lights, each having phases red-orange-green-orange-red-. . . . Construct timed automata TA_i ($i \in \{1, 2\}$) for each of the traffic lights such that the following conditions are satisfied:

- The combination of the lights is safe, i.e., if one light is in a non-red state, then the other light shows red.
- Each green phase is between 30 and 40 seconds long.
- There is a delay between 2 and 3 seconds after the switch from orange to red on the one light, before the switch from red to orange is performed on the other light.
- Each orange phase takes exactly 5 seconds.
- Both lights show green infinitely often.
- TA_1 and TA_2 are symmetrical, only the initial state differs: TA_1 starts in a red state, TA_2 in a green one.

For symmetry reasons you only have write down TA_1.



Exercise 4 (15 points)

The theorem of Knaster & Tarski can be lifted to infinite sets S. Essentially, one replaces $\tau^{|S|}(\varnothing)$ by

$$fp_\tau = \bigcup_{n \in \mathbb{N}} \tau^n(\varnothing).$$

Prove that if $\tau : 2^S \to 2^S$ is monotone then $fp_\tau \subseteq \tau(fp_\tau)$.

We prove that for all $a \in fp_\tau$ we also have $a \in \tau(fp_\tau)$. So let $a \in fp_\tau$. Hence, there is some n such that $a \in \tau^n(\varnothing)$. Since $\tau^0(\varnothing) = \varnothing$ we know that $n > 0$ and thus, $a \in \tau(\tau^{n-1}(\varnothing))$. Since $\tau^{n-1}(\varnothing) \subseteq fp_\tau$ we know by monotonicity of τ that $a \in \tau(\tau^{n-1}(\varnothing)) \subseteq \tau(fp_\tau)$.