First name: _____

Last name: _____

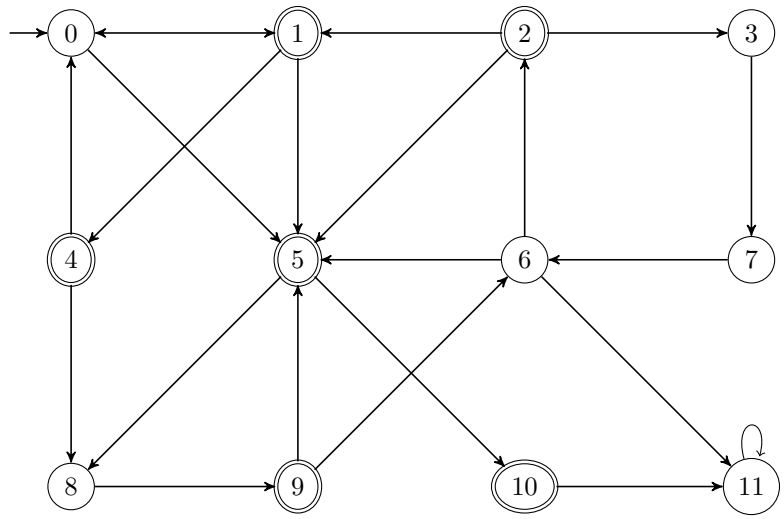
Matriculation number: _____

- Please answer all exercises in a readable and precise way. Please cross out solution attempts which are replaced by another solution.
- Cheating is not allowed. Everyone who is caught will fail the exam.
- Please do not remove the staples of the exam.

Exercise	Maximal points	Points
1	17	
2	31	
3	24	
4	24	
5	24	
Σ	120	
Grade		

Exercise 1 (14 + 3 points)

Consider the following NBA $\mathcal{A}$ (where labels are omitted).



- Fill the table which is obtained when using the algorithm of Yannakakis et. al. where successors are taken in order, i.e., successor i is taken before j iff $i < j$.

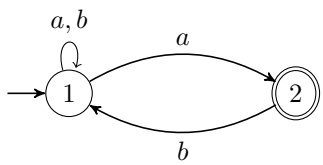
  Here, you should at least write down a new line whenever the set of marked or flagged states are changed.

outer_dfs-stack	**inner_dfs-stack**	**marked**	**flagged**
ε	ε	$\varnothing$	$\varnothing$

- Is there an accepting run of $\mathcal{A}$? If so, which run is extracted by the algorithm?

Exercise 2 (17 + 4 + 3 + 4 + 3 points)

Consider the following NBA $\mathcal{A}$ over $\Sigma = \{a, b\}$.



- Compute the transition profile automaton of $\mathcal{A}$.

- From the lecture we know that the transition profile automaton can be used to compute the equivalence classes of $\sim_{\mathcal{A}}$. Use this connection to determine whether $bbbaaa \sim_{\mathcal{A}} bababa$. If these words are not equivalent w.r.t. $\mathcal{A}$, then provide two states p and q where the run from p to q in $\mathcal{A}$ (possibly via some final state) is only possible with one of the words $bbbaaa$ or $bababa$.

- Let $L = \mathcal{L}(\mathcal{A})$. Let $L' = \Sigma^\omega \setminus L$. Describe L' in your own words.

- Use one of your previous answers to systematically or intuitively construct an automaton $\mathcal{B}$ with $\mathcal{L}(\mathcal{B}) = L'$.

- Provide F1S-formulas for L and L'.

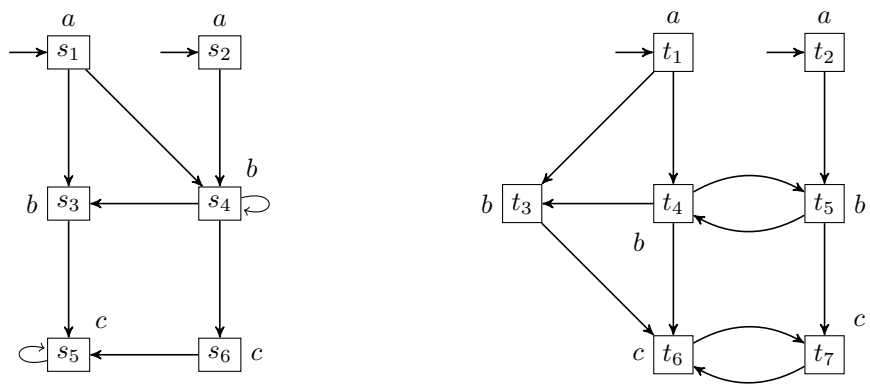
Exercise 3 (24 points)

Explain precisely how one can extract the positional winning strategies of ∃loise and ∀belard from the bottom-up coloring algorithm. Moreover, argue informally why your extracted strategies really are winning strategy.

You only have to consider the winning strategy for ∃loise, since the extraction and argumentation for ∀belard is completely symmetric.

Exercise 4 (12 + 12 points)

Consider the following transition systems.



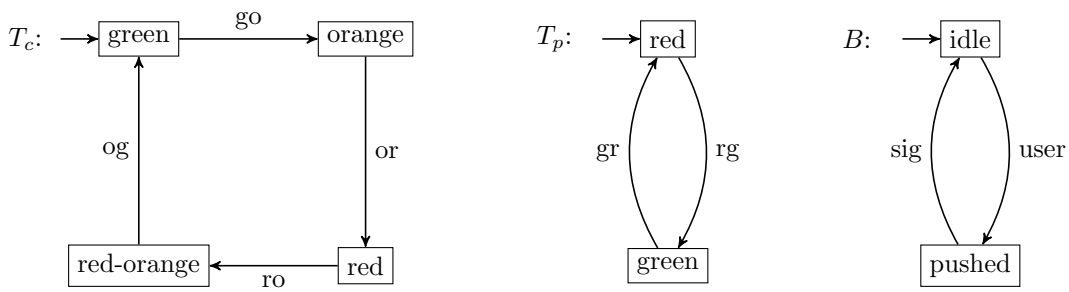
- Use the partition-refinement algorithm to show that these systems are not bisimilar.

- Provide a CTL-formula which distinguishes both systems. Your formula must not use temporal connectives except for EX. You may however use abbreviations for formulas which characterize certain sets of states.

Exercise 5 (10 + 10 + 4 points)

We consider a street with a pedestrian crossing that is operated by two traffic lights: T_c for the cars and T_p for the pedestrians. Moreover, there is a controller C and a button B which can be triggered by a pedestrian via an unsynchronized external action *user*.

The transition systems for the traffic lights and the button are given below:



- Design a controller such that upon the external *user*-action, the following sequence of states is taken (where g stands for green, etc., and where the left entry is for T_c and the right for T_p)

$$(g, r) \to (o, r) \to (r, r) \to (r, g) \to (r, r) \to (ro, r) \to (g, r)$$

- Make a timed automaton out of the controller such that the following requirements are satisfied:
  - The green phase for pedestrians should be 30 seconds long.
  - The switch $(r, r) \to (r, g)$ should be done instantaneously.
  - After green has been turned off for pedestrians, there should be no cars driving in the next 8 seconds.
  - Orange and red-orange phases should be 5 seconds long.
  - The green phase for cars must be at least 45 seconds long in each cycle.

  Try to use as few clocks as possible and prevent time-locks.

- Specify a TCTL-formula describing the requirement that whenever the button was pushed, within 60 seconds the traffic-light for the pedestrians shows green.

  Does your system satisfy this requirement? (just answer yes or no without proof)