This exam consists of four exercises. The available points for each item are written in the margin. You need at least 50 points to pass.

## Explain your answers!

1 Consider the formula $\varphi^{\mathrm{UF}}$

$$
F\left(F\left(x_{1}\right)\right)=G\left(x_{1}, F\left(x_{2}\right)\right) \wedge F\left(F\left(x_{2}\right)\right) \neq G\left(F\left(x_{1}\right), F\left(x_{2}\right)\right) \wedge G\left(x_{1}, x_{2}\right)=F\left(x_{2}\right)
$$

in equality logic with uninterpreted functions.
(a) Use Ackermann's reduction to transform $\varphi^{\mathrm{UF}}$ into an equivalent equality logic formula.
(b) Use Bryant's reduction to transform $\varphi^{\mathrm{UF}}$ into an equivalent equality logic formula.

2 Consider the following equality logic formula $\varphi^{\mathrm{E}}$ :

$$
\begin{gathered}
a=b \wedge a \neq c \wedge(a \neq d \vee e=f \vee g=h) \wedge \\
g=i \wedge h=j \wedge(b=c \vee g \neq i \vee i=j)
\end{gathered}
$$

(a) Compute the equality graph of $\varphi^{\mathrm{E}}$ and list its contradictory cycles.
(b) Compute the propositional skeletion of $\varphi^{\mathrm{E}}$.
(c) Compute a nonpolar chordal equality graph for $\varphi^{\mathrm{E}}$.
(d) Transform $\varphi^{\mathrm{E}}$ into an equisatisfiable propositional formula.
(e) Compute an adequate domain for $\varphi^{\mathrm{E}}$ whose state space is smaller than 10 !

3 Consider the following linear system $S$ over the reals:

$$
\begin{aligned}
2 x_{1}+2 x_{2}+2 x_{3} & \leqslant 2 \\
4 x_{1}-2 x_{2}-x_{3} & \leqslant-3 \\
x_{1}+x_{2} & \geqslant 1
\end{aligned}
$$

[10] (a) Use the generalized simplex method to find a solution for $S$.
(b) Use Fourier-Motzkin variable elimination to find a solution for $S$.
(c) Does $S$ admit any integer solutions?
[20] 4 Group the following concepts in seven categories of four related concepts.


