Automatic Deduction	SS 2010	LVA 703925

EXAM 1 — September 27, 2010

This exam consists of six exercises of which ***you have to select exactly four***. Every exercise is worth 25 points. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers!***

1 Consider the following CNF:

$$\phi = (\neg 1 \vee 5) \wedge (\neg 2 \vee 4) \wedge (\neg 3 \vee \neg 6) \wedge (\neg 3 \vee \neg 5 \vee 7) \wedge (6 \vee \neg 7)$$

[7] (a) Show that the sequence of decisions 1 2 3 (with exhaustive unit propagation after each decision) leads to a conflict.

[9] (b) Formulate the backjump rule of the abstract DPLL framework.

[9] (c) Construct two different backjump clauses and give the corresponding results of applying the backjump rule.

2 Consider the formula φ^{UF}

$$H(F(x_1)) = G(x_1, F(x_2)) \wedge F(H(x_2)) \neq G(F(x_1), F(x_2)) \wedge G(H(x_1), x_2) = H(x_2)$$

in equality logic with uninterpreted functions.

[9] (a) Use Ackermann's reduction to transform φ^{UF} into an equivalent equality logic formula.

[9] (b) Use Bryant's reduction to transform φ^{UF} into an equivalent equality logic formula.

[7] (c) Is φ^{UF} satisfiable?

3 Consider the following equality logic formula φ^{E}:

$$(a = b \vee c = d) \wedge (a = c \vee e = f \vee e \neq j) \wedge b = d \wedge (c \neq d \vee g \neq h) \wedge g \neq i \wedge (h = i \vee f = j)$$

[4] (a) Compute the equality graph of φ^{E} and list its contradictory cycles.

[4] (b) Compute the propositional skeleton of φ^{E}.

[4] (c) Compute a nonpolar chordal equality graph for φ^{E}.

[4] (d) Transform φ^{E} into an equisatisfiable propositional formula.

[9] (e) Compute an adequate domain for φ^{E} whose state space is smaller than 10!

Turn Over **Turn Over** **Turn Over**

4 (a) Determine the satisfiability of the following formulas over the integers by applying Cooper's method:

[10] i. $\exists x.\ 3x < y$

[10] ii. $\forall x.\ 3 < 2x + y \lor x + 2y < 3$

[5] (b) What is the purpose of divisibility predicates in the theory of augmented linear integer arithmetic?

5 Determine the satisfiability of the following formulas over the reals (or rationals) by applying Ferrante and Rackoff's method:

[10] (a) $\exists x.\ 2x = 3y$

[15] (b) $\exists x.\ 3x + 1 < 10 \land 7x - 6 > 8$

6 (a) Determine the satisfiability of the following formulas in the combination of linear arithmetic over the integers and equality logic with uninterpreted functions, using Nelson and Oppen's method:

[10] i. $1 \leqslant x \land x \leqslant 2 \land f(x) \neq f(1) \land f(x) \neq f(2)$

[10] ii. $f(x) = x + y \land x \leqslant y + z \land x + z \leqslant y \land y = 1 \land f(x) \neq f(2)$

[5] (b) What is a stably infinite theory?