Automatic Deduction	SS 2011	LVA 703522

EXAM 1	June 29, 2011

This exam consists of six exercises of which ***you have to select exactly five***. Every exercise is worth 20 points. The available points for each item are written in the margin. You need at least 50 points to pass. ***Explain your answers!***

1 Consider the formula φ^{UF}

$$F(G(x_1)) = G(F(x_1)) \wedge F(G(F(x_2))) = x_1 \wedge F(x_2) = x_1 \wedge G(F(x_1)) \neq x_1$$

in equality logic with uninterpreted functions.

[6] (a) Use Ackermann's reduction to transform φ^{UF} into an equivalent equality logic formula.

[6] (b) Use Bryant's reduction to transform φ^{UF} into an equivalent equality logic formula.

[8] (c) Use the congruence closure algorithm to determine the satisfiability of φ^{UF}.

2 Consider the following equality logic formula φ^{E}:

$$(a = b \vee c \neq d) \wedge (a = c \vee e = f \vee e \neq j) \wedge (b = d \vee g \neq h) \wedge$$
$$(c = d \vee g \neq i) \wedge (i = h \vee f = j) \wedge (g = h \vee e = j)$$

[3] (a) Compute the equality graph of φ^{E} and list its contradictory cycles.

[3] (b) Compute the propositional skeletion of φ^{E}.

[4] (c) Compute a nonpolar chordal equality graph for φ^{E}.

[4] (d) Transform φ^{E} into an equisatisfiable propositional formula.

[6] (e) Compute an adequate domain for $\Phi(E(\varphi^{\mathrm{E}}))$ whose state space is smaller than 10!

[20] **3** Apply Cooper's method to determine the validity of the formula

$$\forall x.\ (\exists y.\ x = 2y) \implies (\exists y.\ 3x = 2y)$$

over the integers.

Turn Over **Turn Over** **Turn Over**

4 Determine the satisfiability of the following formulas over the rationals by applying Ferrante and Rackoff's method:

[10] (a) $\exists x.\ x = 2y \ \wedge\ y < x$

[10] (b) $\forall y.\ 3 < 2x + y \ \vee\ x + 3y < 2$

5 Consider the following satisfiability problem, presented as Yices input:

```
(define x::int)
(define y::int)

(assert (>= (+ x (* 2 y)) 1))
(assert (<= (+ (* 2 x) y) 1))
(assert (<= (* 2 y) 1))
(set-evidence! true)
(check)
```

[4] (a) Name at least two decision procedures that can be used to solve this problem.

[8] (b) Apply one of these procedures to determine satisfiability of the problem.

[8] (c) Determine satisfiability when the variable declarations are changed to

```
(define x::real)
(define y::real)
```

[15] **6** (a) Determine the satisfiability of the formula

$$1 \leqslant x \wedge 2 \leqslant y \wedge f(x) \neq f(1) \wedge f(f(x)) \neq f(f(y)) \wedge f(2) = y \wedge f(x) \neq f(3) \wedge f(2) \leqslant 2 \wedge x \leqslant 3$$

in the combination of linear arithmetic over the integers and equality logic with uninterpreted functions, using Nelson and Oppen's method.

[5] (b) What is a convex theory?