1.     - First observe that append(Us,Vs,Xs) is linear in $|U s|$. Now, let $|X s|=n$. Thus we need $n$ recursive calls to reverse in the proof-tree. For each of these calls at most $n$ calls to append are necessary. Hence the proof-tree contains at most $\mathrm{O}\left(|X s|^{2}\right)$ nodes.

- The following variant of reverse $/ 2$ is linear in length of the first list.

```
        reverse(Xs,Ys) :-
        reverse(Xs,[],Ys).
        reverse([],Ys,Ys).
        reverse([X|Xs],Ys,Zs) :-
        reverse(Xs,[X|Ys],Zs).
```

2. duplicate (Xs,N,Ys) :-
duplicate2 (Xs, N, Ys $\backslash[])$.
duplicate2 ([],_N, Ys $\backslash$ Ys).
duplicate2 ([X|Xs],N,Ys0 $\backslash \mathrm{Ys} 2)$ : -
generate (X, N, Ys0 $\backslash \mathrm{Ys} 1$ ),
duplicate2 (Xs, N, Ys1 $\backslash \mathrm{Ys} 2)$.
generate (_X, $0, Y s \backslash Y s)$.
generate ( $\overline{\mathrm{X}}, \mathrm{N}, \mathrm{Ys} 0 \backslash \mathrm{Ys} 1$ ) :-
$\mathrm{N}>0$,
N 1 is $\mathrm{N}-1$,
generate (X, N1, Ys0 $\backslash[\mathrm{X} \mid \mathrm{Ys} 1])$.
3.     - foo $(\mathrm{X}, \mathrm{Y})$ holds if $Y$ is reachable from $X$ in a graph represented by the predicate edge/2. The graph is traversed breadth-first.

- setof1 (Template,Goal,Set) succeeds with the empty list, if no instance of Template can meet Goal. This is in contrast to the system predicate setof /3, which simply fails in this case. If setof1 $/ 3$ is replaced by setof $/ 3$ in the considered program, then the breadth-first search fails. Let us call the new programm foo'. For example, if we define the following facts:

```
edge(a,b).
edge(a,c).
```

we have that foo ( $\mathrm{a}, \mathrm{c}$ ) holds (as it should), but foo' $(\mathrm{a}, \mathrm{c})$ fails. The meaning of the program changes if setof $1 / 3$ is replaced by the system predicate setof $/ 3$.

Give an example of a goal that succeeds in the original program, but fails in the altered program.
4. We give the complete solution of the problem.

```
complete_knights_tour (N, Knights) :-
    knights(N, Knights), Knights \(=\left[\mathrm{X} /\left.\mathrm{Y}\right|_{\_}\right]\),
    jump (N,X/Y,1/1).
knights (N, Knights) :-
    M is \(\mathrm{N} * \mathrm{~N}-1\),
    knights (N,M,[1/1], Knights).
knights (_, 0, Knights, Knights).
knights(N,M, Visited, Knights) :-
    Visited \(=\left[\mathrm{X} /\left.\mathrm{Y}\right|_{-}\right]\),
    jump (N, X/Y,U/V),
    \(\+\) memberchk(U/V, Visited),
    M1 is M-1,
    knights(N, M1, [U/V| Visited], Knights).
jump (N,A/B,C/D) :-
    jump_dist (X,Y),
    \(C\) is \(A+X, C>0, C=<N\),
    D is \(\mathrm{B}+\mathrm{Y}, \mathrm{D}>0, \mathrm{D}=<\mathrm{N}\).
jump_dist (1,2).
jump_dist \((2,1)\).
jump_dist \((2,-1)\).
jump_dist \((1,-2)\).
jump_dist \((-1,-2)\).
jump_dist \((-2,-1)\).
jump_dist \((-2,1)\).
jump_dist \((-1,2)\).
```

A rule is a universally quantified logical formula of the form $A \leftarrow B_{1}, B_{2}, \ldots, B_{n}$, where $A$ is a goal and for all $i=1, \ldots, n$ : $B_{i}$ is a goal.
An SLD-refutation is a finite SLD-derivation ending in the goal to be proven.

Logic programming is a declarative programming paradigm, that is, the computation of a function is made a first-class citizen.

The declarative semantics of a program $P$ is the minimal model of $P$.

The order of goals is irrelevant in the computation model of logic programming, but not the order of rules.

The order of goals and the order of rules is irrelevant in the computation model of Prolog.
Prolog is a language without types and the main technique to manipulate data is unification.
Difference lists are ineffective if the generation of different sections of a list depend on each other.

A meta-interpreter in Prolog interprets Prolog terms on the Warren abstract machine.
The predicate bagof (Template, Goal, Bag) unifies Bag with the alternatives of
$\square$
$\square$


