Computational Logic

Automatic Proofs in Equational Logic

Status Report

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May 9, 2012

Outline

- Objective
- Preliminaries
- Recording Completion
- Implementation
- Live-Demo
- Current State
- Résumé

Objective

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deduce $\dfrac{(E, R)}{(E \cup \{s \approx t\}, R)}$ if $s \leftarrow u \rightarrow t$

delete $\dfrac{(E \cup \{s \approx s\}, R)}{(E, R)}$

orient $\dfrac{(E \cup \{s \mathrel{\dot{\approx}} t\}, R)}{(E, R \cup \{s \rightarrow t\})}$ if $s > t$

compose $\dfrac{(E, R \cup \{s \rightarrow t\})}{(E, R \cup \{s \rightarrow u\})}$ if $t \rightarrow u$

simplify $\dfrac{(E \cup \{s \mathrel{\dot{\approx}} t\}, R)}{(E \cup \{u \mathrel{\dot{\approx}} t\}, R)}$ if $s \rightarrow u$

collapse $\dfrac{(E, R \cup \{s \rightarrow t\})}{(E \cup \{u \approx t\}, R)}$ if $s \overset{!}{\rightarrow} u$

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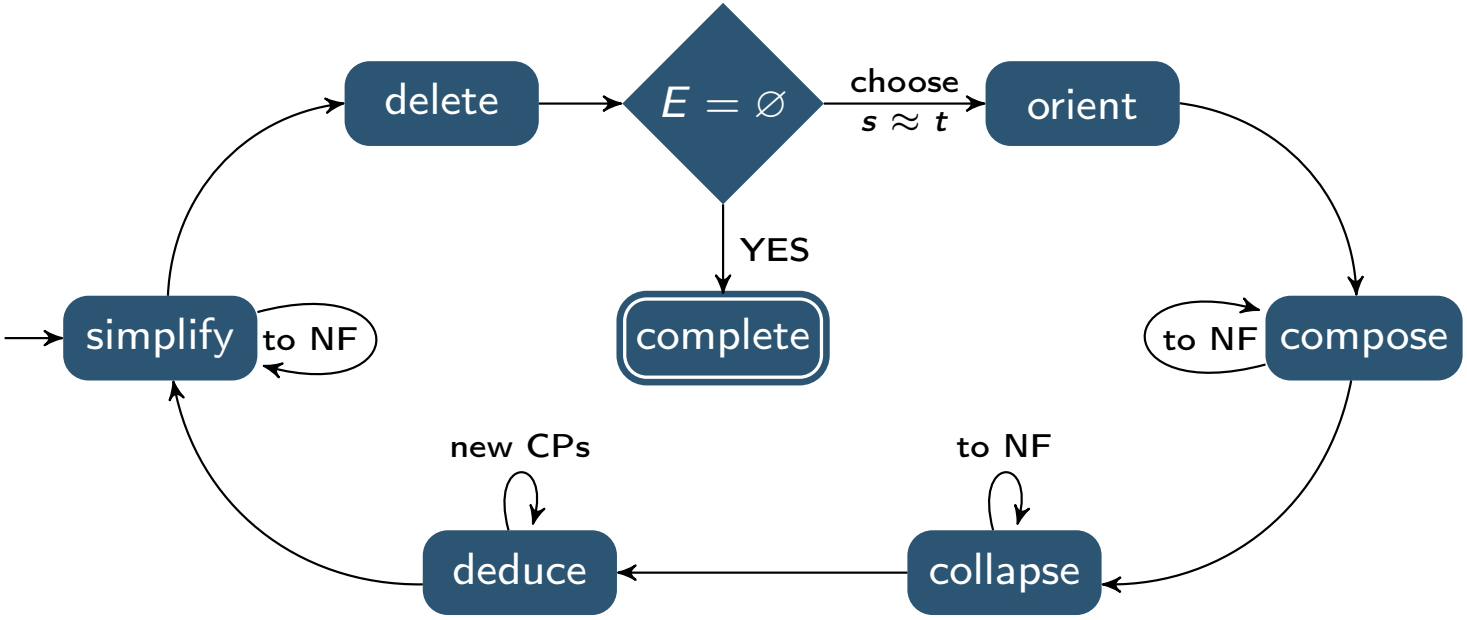

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[r] reflexivity

$$\frac{}{t \approx t}$$

[s] symmetry

$$\frac{s \approx t}{t \approx s}$$

[t] transitivity

$$\frac{s \approx t, t \approx u}{s \approx u}$$

[a] application

$$\frac{}{l\sigma \approx r\sigma}$$

[c] congruence

$$\frac{s_1 \approx t_1, \ldots, s_n \approx t_n}{f(s_1, \ldots, s_n) \approx f(t_1, \ldots, t_n)}$$

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Example

$$E = \{\mathsf{ff} \approx \mathsf{f}, \mathsf{ggf} \approx \mathsf{g}\}$$
$$E \vdash \mathsf{fgf} \approx \mathsf{fgg}$$

$$\dfrac{\dfrac{\dfrac{}{\mathsf{ggff} \approx \mathsf{gf}}\,[\mathsf{a}]}{\mathsf{fggff} \approx \mathsf{fgf}}\,[\mathsf{c}]}{\mathsf{fgf} \approx \mathsf{fggff}}\,[\mathsf{s}] \qquad \dfrac{\dfrac{\dfrac{\mathsf{ff} \approx \mathsf{f}}{\vdots}\,[\mathsf{c}]}{\mathsf{fggff} \approx \mathsf{fggf}}\,[\mathsf{c}] \qquad \dfrac{\vdots \quad \vdots}{\mathsf{fggf} \approx \mathsf{fgg}}\,[\mathsf{t}]}{\mathsf{fggff} \approx \mathsf{fgg}}\,[\mathsf{t}]$$
$$\mathsf{fgf} \approx \mathsf{fgg}\,[\mathsf{t}]$$

Recording Completion

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deduce

$$\frac{(E, R, H)}{(E \cup \{m : s \approx t\}, R, H \cup \{m : s \overset{j}{\leftarrow} u \overset{k}{\rightarrow} t\})} \quad \text{if } s \overset{j}{\leftarrow} u \overset{k}{\rightarrow} t$$

orient$_l$

$$\frac{(E \cup \{i : s \mathrel{\dot{\approx}} t\}, R, H)}{(E, R \cup \{i : s \rightarrow t\}, H)} \quad \text{if } s > t$$

orient$_r$

$$\frac{(E \cup \{i : s \approx t\}, R, H \cup \{i : s \overset{j}{\circ_1} u \overset{k}{\circ_2} t\})}{(E, R \cup \{i : t \rightarrow s\}, H \cup \{i : t \overset{k}{(\circ_2)}^{-1} u \overset{j}{(\circ_1)}^{-1} s\})} \quad \text{if } t > s$$

simplify

$$\frac{(E \cup \{i : s \mathrel{\dot{\approx}} t\}, R, H)}{(E \cup \{m : u \mathrel{\dot{\approx}} t\}, R, H \cup \{m : u \overset{l}{\leftarrow} s \overset{i}{\rightarrow} t\})} \quad \text{if } s \overset{l}{\rightarrow} u$$

delete

$$\frac{(E \cup \{i : s \approx s\}, R, H \cup \{i : s \circ_1 v \circ_2 s\})}{(E, R, H)}$$

compose

$$\frac{(E, R \cup \{i : s \rightarrow t\}, H)}{(E, R \cup \{m : s \rightarrow u\}, H \cup \{m : s \overset{i}{\rightarrow} t \overset{j}{\rightarrow} u\})} \quad \text{if } t \overset{j}{\rightarrow} u$$

collapse

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Recording Completion

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Record → Compare → Recall → Plant & Grow

Implementation

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KBCV 1.0

$\sim$4100 LOC

Implementation

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~4100 LOC

termlib 1.2

~1700 LOC

- Indices
- Recording completion
- Performance optimization

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- Enhanced automatic completion
- Equational logic proofs
- Certifiable output

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	KBCV	MAXCOMP	MKBTT	Slothrop
completed	86	86	81	71
`LS94_P1`	✓			
`SK90_3.26`	✓			

Table: Experimental results on 115 systems.

Live-Demo

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Completion	$E \vdash s \approx t$	
successful	yes	✓

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Completion	$E \vdash s \approx t$	
successful	yes	✓
successful	no	✓

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Completion	$E \vdash s \approx t$	
successful	yes	✓
successful	no	✓
not successful	yes	$\sim$

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Completion	$E \vdash s \approx t$	
successful	yes	✓
successful	no	✓
not successful	yes	$\sim$
not successful	no	×

Résumé

- Assignment: Automatic proofs in equational logic.
- Basics: Completion and equational logic.
- Recording completion.
- Implementation in KBCV.
- Demo.
- Current state.

More Information

Visualizing Knuth-Bendix Completion

Thomas Sternagel

Bachelor Thesis, University of Innsbruck, 2010.

Automatic Proofs in Equational Logic

Thomas Sternagel

Master Seminar Report, University of Innsbruck, 2010.

Automatic Proofs in Equational Logic (2) - Model Finding

Thomas Sternagel

Master Seminar Report, University of Innsbruck, 2011.

KBCV- Knuth-Bendix completion visualizer

Thomas Sternagel and Harald Zankl

System Description, IJCAR 2012, LNAI, 2012. To appear.

Recording completion for finding and certifying proofs in equational logic

Thomas Sternagel, René Thiemann, Harald Zankl, Christian Sternagel

IWC 2012, 2012. To appear.

Feedback

»This tool will be of interest to all students and users of completion.«

Reviewer X

»I downloaded and installed KBCV and found it a pleasure to use. The nice graphical user interface is intuitive and useful for experimentation.«

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The talk is complete now!!!

Thank you for your attention!

Any questions?