First Exam
Automated Reasoning, LVA 703607

July 5, 2013

Name: **Studentnumber**:

The exam consists of 5 exercises with a total of 100 points. Please fill our your name and credentials *before* you start the exam.

1	2	3	4	5	Sum

0–49:	**5**	50–59:	**4**	60–74:	**3**	75–89:	**2**	90–100:	**1**

1. Consider the clause set:

$$\mathcal{C} = \{\mathsf{P}(x) \vee \mathsf{Q}(\mathsf{f}(\mathsf{a})), \neg\mathsf{P}(y) \vee \mathsf{Q}(y), \mathsf{P}(\mathsf{f}(v)) \vee \neg\mathsf{Q}(w), \neg\mathsf{P}(z) \vee \neg\mathsf{Q}(\mathsf{f}(\mathsf{a}))\} ,$$

   where a (f) is an individual (function) constant. Give a closed semantic tree for $\mathcal{C}$. (10 pts)

2. Show completeness of unrestricted first-order tableaux.

   Hint: It suffices to sketch the argument and in particular the model existence theorem can be employed. (25 pts)

3. Consider variants of Skolemisation and let A denote a rectified formula in negation normal form.

   a) Define the (refutational) structural Skolem form of A. (10 pts)

   b) Define the *prenex Skolem form* of A. (5 pts)

   c) What is the *computational* difference between these two variants of Skolemisation? (5 pts)

4. Prove the following lemma:

   Lemma. Let τ be a ground substitutions and consider the following ground factoring step:

$$\frac{C\tau \vee A\tau \vee B\tau}{C\tau \vee A\tau} \quad ,$$

   where $A\tau = B\tau$. Then there exists a mgu σ, such that σ is more general then τ and the following factoring step is valid:

$$\frac{C \vee A \vee B}{(C \vee A)\sigma} \quad .$$

   Hint: A slightly more concrete proof than in the lecture notes is expected. (25 pts)

5. Determine whether the following statements are true or false. Every correct answer is worth 2 points; wrong answers "earn" -1 points. (20 pts)

statement	**yes**	**no**
Let $\mathcal{G}$ denote a set of universal formulas. Then $\mathcal{G}$ is satisfiable iff $\mathcal{G}$ has a Herbrand model.	☐	☐
The DPLL-method includes a) the tautology rule, b) the pure literal rule, c) the splitting rule, and d) the zero-one rule.	☐	☐
Optimised Skolemisation is an example of an inner Skolemisation technique.	☐	☐
Strong Skolemisation is an example of an outer Skolemisation technique.	☐	☐
A literal L is strictly maximal if there exists a ground substitution σ such that for no other literal M: $M\sigma \succ_{\mathsf{L}} L\sigma$ holds.	☐	☐
If $\mathcal{C}$ is the clause representation of the (negated) word problem $\mathcal{E} \models s = t$, then the saturation of $\mathcal{C}$ with respect to superposition (and equality resolution) contains $\Box$ iff $\mathcal{E} \models s = t$.	☐	☐
A clause $C \vee s = t$ is reductive if $s \succ t$ and $s = t$ is maximal with respect to C.	☐	☐
A reductive conditional rewrite system is confluent only if all critical pairs converge.	☐	☐
A candidate model is a model.	☐	☐
All non-redundant superposition inferences are liftable.	☐	☐