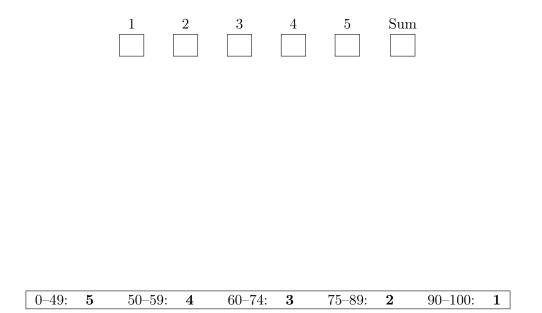
## First Exam Automated Reasoning, LVA 703607

July 5, 2013

Name:

Studentnumber:

The exam consists of 5 exercises with a total of 100 points. Please fill our your name and credentials *before* you start the exam.



1. Consider the clause set:

 $\mathcal{C} = \{\mathsf{P}(x) \lor \mathsf{Q}(\mathsf{f}(\mathsf{a})), \neg \mathsf{P}(y) \lor \mathsf{Q}(y), \mathsf{P}(\mathsf{f}(v)) \lor \neg \mathsf{Q}(w), \neg \mathsf{P}(z) \lor \neg \mathsf{Q}(\mathsf{f}(\mathsf{a}))\} ,$ 

where a(f) is an individual (function) constant. Give a closed semantic tree for C.

(10 pts)

2. Show completeness of unrestricted first-order tableaux.

*Hint*: It suffices to sketch the argument and in particular the model existence theorem can be employed. (25 pts)

- 3. Consider variants of Skolemisation and let A denote a rectified formula in negation normal form.
  - a) Define the (refutational) structural Skolem form of A. (10 pts)
  - b) Define the prenex Skolem form of A. (5 pts)
  - c) What is the *computational* difference between these two variants of Skolemisation? (5 pts)
- 4. Prove the following lemma:

**Lemma.** Let  $\tau$  be a ground substitutions and consider the following ground factoring step:

$$\frac{C\tau \vee A\tau \vee B\tau}{C\tau \vee A\tau} \,,$$

where  $A\tau = B\tau$ . Then there exists a mgu  $\sigma$ , such that  $\sigma$  is more general then  $\tau$  and the following factoring step is valid:

$$\frac{C \lor A \lor B}{(C \lor A)\sigma} \,.$$

*Hint*: A slightly more concrete proof than in the lecture notes is expected. (25 pts)

5. Determine whether the following statements are true or false. Every correct answer is worth 2 points; wrong answers "earn" -1 points.	(20	0 pts)
statement	yes	no
Let $\mathcal G$ denote a set of universal formulas. Then $\mathcal G$ is satisfiable iff $\mathcal G$ has a Herbrand model.		
The DPLL-method includes a) the tautology rule, b) the pure literal rule, c) the splitting rule, and d) the zero-one rule.		
Optimised Skolemisation is an example of an inner Skolemisation technique.		
Strong Skolemisation is an example of an outer Skolemisation technique.		
A literal $L$ is strictly maximal if there exists a ground substitution $\sigma$ such that for no other literal $M: M\sigma \succ_{L} L\sigma$ holds.		
If $C$ is the clause representation of the (negated) word problem $\mathcal{E} \models s = t$ , then the saturation of $C$ with respect to superposition (and equality resolution) contains $\Box$ iff $\mathcal{E} \models s = t$ .		
A clause $C \lor s = t$ is reductive if $s \succ t$ and $s = t$ is maximal with respect to $C$ .		
A reductive conditional rewrite system is confluent only if all critical pairs con- verge.		
A candidate model is a model.		
All non-redundant superposition inferences are liftable.		