Homework

1. Prove in HOL-Light using tactics:

   a) `A \/ B ==> B \/ A`

   b) `(A ==> B ==> C) ==> (A ==> B) ==> A ==> C`

   c) `(!x. P x ==> Q x) ==> ((?y. Q y) ==> P a) ==> Q b ==> Q (a : A)`

   Without using automated tactics. Hint: look for the disjunction and universal quantification related tactics in `VERY_QUICK_REFERENCE.txt`

2. Prove the Church-Rosser property of untyped λ-calculus

3. Find the typing derivations of the following λ_P terms

   (a)
   $$X : \star,\, x : X \;\vdash\; x :?$$

   (b)
   $$X : \star \vdash (X \to X) :?$$

   (c)
   $$A : \star,\; P : A \to \star,\; a : A \vdash (P\,a) \to \star :?$$

   (d)
   $$\alpha : 0 \to \star, \beta : 0 \to \star \vdash \lambda y : (\forall x : 0.\alpha x \to \beta x).\lambda z : (\forall x : 0.\alpha x).\lambda x : 0.yx(zx) :?$$

   (e)
   $$\alpha : 0 \Rightarrow \star \vdash (\Pi y : 0)(\alpha y \Rightarrow \star) :?$$

Remark: 0 is a special type variable, which we will get to know in the next lecture. You can assume an additional assumption $0 : \star$.