



## Homework

1. Define recursively in HOL-Light the function  $f : \mathbb{N} \rightarrow \mathbb{N}$  that returns the sum of  $n$  first natural numbers.
  - (a) Using `new_recursive_definition` and `num_RECURSION`.
  - (b) Using `new_definition`. Hint: only one equation is allowed, and you need to encode the recursion yourself (inventing something like the  $Y$  combinator or maybe something simpler).
2. Prove that  $fn = \frac{n(n+1)}{2}$ . Use `DIV` for natural division.
  - (a) Using `INDUCT_TAC` and `ARITH...`
  - (b) Only using basic tactics, matching and rewriting
3. Prove in Gentzen-style natural deduction:
  - (a)  $(\forall y. Qy \rightarrow Py) \rightarrow Qa \rightarrow Pa$
  - (b)  $(\forall x. \forall y. Px \rightarrow Py) \rightarrow Pa \rightarrow \forall x. Px$
4. Give  $\lambda_P$  derivations corresponding to the above proofs.