Minlog

Interactive Theorem Proving

Thomas Sternagel

Computational Logic
Institute of Computer Science
University of Innsbruck

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Outline

- Basic Facts
- Features
- Minimal logic
  - $\Rightarrow$ Intuitionistic logic
  - $\Rightarrow$ Classical logic
- Program extraction

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- Minlog live-demo

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- Proof style: Goal driven reasoning (“backward chaining”)
- Documentation:
  - Tutorial [1]
  - Reference manual [2]
  - Minimal Logic for Computable Functions [3]

Features I - From the manual

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  - Predicate variables for comprehension terms
- Terms with same NF are identified

Features II - From “The Seventeen Provers of the World”

proof assistent	Minlog	HOL	Mizar	Isabelle/Isar
de Bruijn criterion	+	+	−	+
Poincaré principle	+	+	−	+
extensible/programmable by user	+	+	−	+
powerful automation	−	+	−	+
readable proof input files	−	−	+	+
constructive logic supported	+	−	−	+
logical framework	−	−	−	+
typed	+	+	+	+
decidable types	+	+	+	+
dependent types	−	−	+	−
based on HOL	−	+	−	+
based on ZFC	−	−	+	+
large math stdlib	−	+	+	+
statement about $\mathbb{R}$	+	+	+	+
statement about $\surd$	−	+	+	+

Minimal Logic [5, 4, 6]

FOL language:

$$
\begin{aligned}
f &:= a \mid f \to f \mid \forall x.f \mid f \vee f \mid f \wedge f \mid \exists x.f \\
a &:= \bot \mid R\vec{t} \\
t &:= x \mid f\vec{t}
\end{aligned}
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A derivation is the formal representation of a proof
Derivable in minimal logic:

$$\Gamma \vdash A$$

Minimal Logic I

$$u : A$$

$$\dfrac{\begin{matrix}[u\colon A] \\ \mid M \\ B\end{matrix}}{A \to B} \to^+ u \qquad\qquad \dfrac{\begin{matrix}\mid M \\ A \to B\end{matrix} \qquad \begin{matrix}\mid N \\ A\end{matrix}}{B} \to^-$$

$$\dfrac{\begin{matrix}\mid M \\ A\end{matrix}}{\forall x.A} \forall^+ x \qquad\qquad \dfrac{\begin{matrix}\mid M \\ \forall x.A(x)\end{matrix} \qquad t}{A(t)} \forall^-$$

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$$\frac{\begin{matrix}[u : A]\\ \mid M\\ B\end{matrix}}{A \to B}\to^{+} u \qquad \frac{\begin{matrix}\mid M\\ A \to B\end{matrix} \qquad \begin{matrix}\mid N\\ A\end{matrix}}{B}\to^{-}$$

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$$\frac{\begin{matrix}\mid M\\ A\end{matrix}}{A \vee B}\vee_{0}^{+} \qquad \frac{\begin{matrix}\mid M\\ B\end{matrix}}{A \vee B}\vee_{1}^{+} \qquad \frac{\begin{matrix}\mid M\\ A \vee B\end{matrix} \qquad \begin{matrix}[u : A]\\ \mid N\\ C\end{matrix} \qquad \begin{matrix}[v : B]\\ \mid K\\ C\end{matrix}}{C}\vee^{-} u, v$$

$$\frac{\begin{matrix}\mid M\\ A\end{matrix} \qquad \begin{matrix}\mid N\\ B\end{matrix}}{A \wedge B}\wedge^{+} \qquad \frac{\begin{matrix}\mid M\\ A \wedge B\end{matrix} \qquad \begin{matrix}[u : A]\ [v : B]\\ \mid N\\ C\end{matrix}}{C}\wedge^{-} u, v$$

$$\frac{t \qquad \begin{matrix}\mid M\\ A(t)\end{matrix}}{\exists x.A(x)}\exists^{+} \qquad \frac{\begin{matrix}\mid M\\ \exists x.A\end{matrix} \qquad \begin{matrix}[u : A]\\ \mid N\\ B\end{matrix}}{B}\exists^{-} x, u$$

Minimal Logic II

Negation:

$$\neg A \equiv A \rightarrow \bot$$

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Derivable:

$$A \rightarrow \neg\neg A$$
$$\neg\neg\neg A \rightarrow \neg A$$

Negation:

$$\neg A \equiv A \to \bot$$

Derivable:

$$A \to \neg\neg A$$

$$\neg\neg\neg A \to \neg A$$

In general not derivable:

$$\neg\neg A \to A$$

Intuitionistic / Classical Logic

classical (weak) $\tilde{\vee}$,$\tilde{\exists}$:

$$A \tilde{\vee} B \equiv \neg A \to \neg B \to \bot$$

$$\tilde{\exists} x.A \equiv \neg \forall x.\neg A$$

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Ex falso quodlibet:

$$\bot \to A \qquad \text{(Efq)}$$

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Stability:

$$\neg\neg A \to A \tag{Stab}$$

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$$\Gamma \cup \text{Efq} \vdash A \Rightarrow \Gamma \vdash_i A$$
$$\Gamma \cup \text{Stab} \vdash A \Rightarrow \Gamma \vdash_c A$$

Program Extraction

- Program Extraction from constructive proofs
  - Realizability for constructive existence proofs
- Program Extraction from classical proofs
  - Refined A-translation [7]
  - Gödel's Dialectica Interpretation

Bibliography I

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Thank you for your attention!
Any questions?