

Interactive Theorem Proving

Week 9

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Summary

So far

Proof Assistants, HOL Light, λ_{\rightarrow} , λ_P , λ_2 , Curry-Howard

- Mizar Project
- Foundations
- Natural deduction

Today: Mizar

- Syntax
- Types
- Adjectives
- Checker
- Foundations

27 special symbols

&
c=

110 reserved words

contradiction
not
or
implies
iff
for x holds a(x)
ex x st a(x)

Mizar Types

A type hierarchy

- Function of X,Y
- PartFunc of X,Y
- Relation of X,Y
- Subset of [:X,Y:]
- set

Adjectives

- Examples
 - one-to-one Function of X,Y
 - finite non-empty proper Subset of X
- Automatic deriving of type information using registrations
- Overloading of notations
- Types must be non-empty

No set of inference rules

“obviousness w.r.t. an algorithm” by M. Davis

de Bruijn criterion is not preserved

- new computation mechanisms (CAS, DS)
- more automation in the equality calculus
- more general statements in an inference

Implementation

- Separate processes
- Parser
 - Scanner
 - Environment
 - Operator syntax
- Analyzer
 - Identification of Concepts
 - Disambiguation of Notations
 - **Types!** (adjectives)
- Reasoner - Structure of the proof
- Checker
 - Prechecker - propositional calculus
 - Equalizer - equational calculus
 - Unifier
- Post-processing
 - Relprem, Relinfer, ...

Congruence closure

Is $x = y$ a consequence of $y = z$, $f(y) = z$, and $f(z) = x$?

- Monotonicity
- Transitivity
- Symmetry

Implementation

Equalizer

- Disprover processes all disjuncts (DNF)
- Forms of a term
- Congruence closure merges adequate EQ-classes
- Reports contradictions
- Prepares input for the Unifier

Sources of equalities

- Explicit
- Term expansions (equals)
- Properties
- Term reductions
- Term identifications
- Processing structures
- Reconsider

MML and its revisions

Biggest library of formal mathematics

- 1177 articles
- 52775 theorems
- 10670 definitions
- 820 schemes
- 11333 registrations

Revisions

- Access to Mizar source
- New articles
- Things becoming obvious

- Hidden
- Tarski
- Boole
- Tuple
- Relation
- Function

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Next time

- Deep vs Shallow Embedding
- Logical Frameworks