Solutions

This exam consists of 5 exercises. Explain your answers. You can reach 100 points and need 50 to pass.

1. Consider a graph $G = (V, E)$.

(9) (a) Mention three possibilities to encode reachability in SAT.

Solution. (i) Zeno of Elea, (ii) Floyd-Warshall, (iii) adjacency matrices.

(b) For **one** of these encodings:

(4) i. Write down the encoding.

(5) ii. Determine the number of variables (by means of O notation).

(5) iii. Determine the size of the encoding (by means of O notation).

(22) **2.** Consider the PB constraint $7x_1 + 5x_2 + 3x_3 \leq 11$. Compute the (interval) BDD representation (using the descending variable order).

Solution. We only give the interval for the root node: x_1 has interval $[10, 14]$.

(11) **3.** (a) Purify the NIA constraint $c(ac + bc) \leq abc$.

Solution. Introducing fresh variables for all expressions yields:

$$\underbrace{(x_4 \leq x_5) \wedge (x_3 = x_1 + x_2)}_{\text{linear}} \wedge \underbrace{(x_1 = ac) \wedge (x_2 = bc) \wedge (x_4 = cx_3) \wedge (x_5 = abc)}_{\text{non-linear}}$$

(11) (b) Linearize the pure non-linear constraint $(0 \leq a \leq 2) \wedge (0 \leq b \leq 2) \wedge (x = cba)$.

Solution. We first split on a then on b. (Note that the algorithm from the lecture introduces variables to avoid an exponential blowup).

$$\begin{aligned}
&(a = 0) \rightarrow (x = 0) \wedge \\
&(a = 1) \rightarrow (((b = 0) \rightarrow (x = 0)) \wedge ((b = 1) \rightarrow (x = c)) \wedge ((b = 2) \rightarrow (x = 2c))) \wedge \\
&(a = 2) \rightarrow (((b = 0) \rightarrow (x = 0)) \wedge ((b = 1) \rightarrow (x = 2c)) \wedge ((b = 2) \rightarrow (x = 4c)))
\end{aligned}$$

4. Consider the sets
$M_1 = \{\texttt{studied}, \texttt{prepared}, \texttt{passed}\}$,
$M_2 = \{\texttt{studied}, \texttt{prepared}, \texttt{blackout}\}$,
$M_3 = \{\texttt{amnesic}\}$,
and the ASP P:

```
passed ← prepared, not blackout
prepared ← studied, not amnesic
studied
```

For $1 \leq i \leq 3$:

(12) (a) Compute the reduct of P wrt. M_i.

(9) (b) Determine if M_i is an answer set of P.

Solution.

	reduct	answer set
M_1	`passed` $\leftarrow$ `prepared`, `prepared` $\leftarrow$ `studied`, `studied`	✓
M_2	`prepared` $\leftarrow$ `studied`, `studied`	✗
M_3	`passed` $\leftarrow$ `prepared`, `studied`	✗

(12) **5.** Indicate for each of these statements if they are true or false.
(Correct answer: 2 points – wrong answer: 0 points)

(a) For every propositional formula there is an equivalent CNF.

Solution. True

(b) For every propositional formula Tseitin's transformation produces more clauses than Greenbaum&Plaisted's transformation.

Solution. False

(c) For every PBC the (interval) BDD is polynomial.

Solution. False

(d) For every NIA formula there is an equivalent LIA formula.

Solution. False

(e) For every bitvector formula there is an equivalent CNF.

Solution. True

(f) Bit-blasting NIA to CNF is sound and complete.

Solution. False