History Of The Boolean Algebra

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1 Introduction

In this short article I want to look back at the history of the Boolean algebra. First I will to start with a short introduction of George Boole and take a closer look at his book *The Mathematical Analysis Of Logic*. Then I will show some early contributions made to the Boolean algebra by Augustus De Morgan and Charles Sanders Peirce. After this I will give a short overview of how the notation of the Boolean algebra changed over time. I also want to show the present day usage of the Boolean algebra by looking at the work of Claude Elwood Shannon and giving a short example of Model Checking.

2 Boolean Algebra

2.1 Motivation

As I started learning about the Boolean algebra I encountered two different types of notation for the same algebra. I always wondered why this is the case. The only way to find out how this came to be is to look at the history of this algebra. Since I use the Boolean algebra rather often in my studies in Computer Science it is also a good thing to know a little about the history and not only use it blindly. So let me start with a short introduction of the man who gave the Boolean algebra its name: *George Boole*.

2.2 George Boole

George Boole was born on the 2. November 1864 in Lincoln, England as the child of the tradesman John Boole. As he had only an elementary School education most of his expertise were either self-thought or achieved throught the help of his fathers friends. At the age of 16 he taught in Heigham's School in Doncaster, and participated in the Lincoln Mechanics' Institution. At age 19 he established his own school in Lincoln, which he had for four years before he took over Hall's Academy. In 1940 he came back to Lincoln, where he started a boarding school. In 1849 he was appointet the first professor of mathematics at Queen's College in Cork, Ireland. There Boole published papers on

Differntial equations, Analysis and symolic logic. With his work *The Mathematical Analysis Of Logic* he lay the cornerstone for the Boolean algebra as we know it today. Boole died on 8. December 1864 in Balintemple, Ireland.

2.3 The Mathematical Analysis Of Logic

As this is only a short overview of the Boolean algebra, there simply is no place to look completely at his extensiv work. So we will only look only at a small part.
It wasn't Booles intention to create the Boolean algebra, but rather to construct a philosophical language and to establish a Calculus of Logic. He himself wasn't sure if his work would be recognized:

> What may be the final estimate of the value of the system, I have neither the wish nor the right to anticipate[1].

Like in every Calculus there also has to be a set of rules or laws in the Logic Calculus. The three laws established by Boole were the following:

$$x(u+v) = xu + xv \tag{1}$$
$$xy = yx \tag{2}$$
$$x^n = x \tag{3}$$

These three laws are the law of distribution (1), the commutativ law (2) and the idempotence as we know them today (3).

2.4 Contributors

The Boolean algebra wasn't created by Boole alone. There were many contributions in the years up to the present time. In this section I want to show the most famous contributions of Augustus De Morgan and Charles Sanders Peirce.

2.4.1 Augustus De Morgan

Augustus De Morgans first work regarding the Boolean algebra, or to be exact regarding the *Formal Logic*[2] was released around the same time as Booles The Mathematical Analysis Of Logic. This is, because even before Boole and De Morgan released their books they were, together with Sir William Rowan Hamilton, discussing the topic of Logic via letters. His most famous contribution were the De Morgan's laws :

$$\neg(P \wedge Q) \Leftrightarrow (\neg P) \vee (\neg Q) \tag{4}$$
$$\neg(P \vee Q) \Leftrightarrow (\neg P) \wedge (\neg Q) \tag{5}$$

2.4.2 Charles Sanders Peirce

Peirce first article regarding the Boolean algebra was released in the year 1986. He proved that by using the one of the operations NAND ('↑' called Sheffer stroke after Henry Maurice Sheffer, and NOR ('↓' called Quine dagger after Willard Van Orman Quine or the by Peirce used 'ampheck' '⅄' also known as Peirce arrow) every other operation can be simulated. Peirce also introduced a standard notation for the First-order predicate logic and worked in the field of semiotiks[1].

3 Present Day Boolean Algebra

3.1 The Notation

Let me start this chapter with how the Boolean algebra came to it's current form. In 1936 Marshall Harvey Stone united all the work done regarding the Boolean algebra under the name *Boolean Ring*. The Boolean algebra (A,∨,∧,¬,1,0) and the Boolean ring (A,+,*,-,1,0) are isomorphic[2]. Meaning:

$$x + y = x \vee y \tag{6}$$
$$x * y = x \wedge y \tag{7}$$
$$-x = \neg x \tag{8}$$

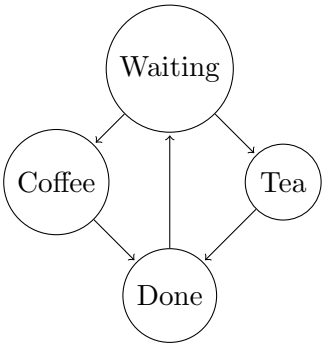
So the way from Booles first publication to Boolean algebra as we know it today took almost 100 years. To give a short overview:
In 1877 Ernst Schröder created the first formal Axiomatic system for the Boolean algebra and Guiseppe Peaon brought it in the form we know today. The '∨' symbol was created in 1906 by Bertrand Russell and the '¬', and '∧' symbols were created by Arend Heyting in 1930. The name Boolean algebra was first used by Henry Maurice Sheffer in 1913. The missing piece for the Boolean algebra to be a ring, the EXCLUSIVE-OR '⊕' was added by Ivan Ivanovich Zegalkin in 1927.

3.2 Claude Elwood Shannon

Shannon proved that Boolean algebra could be used to simplify electro mechanical relays. An expansion of this work was his prove that Boolean algebra problems could be solved by arrangements of relays, which is the basic concept of all electronic digital computers. He also was the first to use Boolean algebra to construct digital circuits, which is still used today to construct new CPUs and other PC parts.

[1] Theory of signs and Theory of meaning

[2] On my research of the history of the Boolean algebra I found different usages of the above mentioned notations. And found the above used to be most common in the english literature.



The graph to the left represents a simple beverage dispenser. The dispenser is waiting for a job (Waiting), which can be either to make coffee (Coffee) or to make tea(Tea). When he is done he waits for someone to take the drink (Done). After this he goes back to waiting for the next job (Waiting).

Figure 1: simple beverage dispenser

3.3 Model Checking

The Boolean algebra is not only used in electronic engineering. Another interesting use is *Model Checking*. In model checking a Problem (e.g. a computer program) is represented as a model $\mathcal{M}$ and the specification as a formula ϕ. And if $\mathcal{M}$ satisfies ϕ the program is correct. Different kinds of logics are used in model checking. I want to look at the *Linear Time Temporal Logic* or short *LTL* and give a short example for how it works.

3.3.1 Semantics

In LTL we look at an infinite path (in the form of a formula) and try to find out if it evaluates to true. Lets try this with the beverage dispenser graph above. If we consider the path: Waiting $\rightarrow$ Coffee $\rightarrow$ Done $\rightarrow$ Waiting $\rightarrow \cdots$. We can check for different things like: Does the state after 'waiting' has the property 'coffee'? As this should only be a small explanation I will stop at here.

4 Summary

I hope to have given a clear overview of the history of the Boolean algebra. As mentioned before due to the shortness of this article I was only able to cover very little of the different subjects that came up in this article. Everyone who wants to know more about the subject I encourage to read the work of Boole and De Morgan. Because of the old english used they are not easy to read, but interesting.

References

[1] George Boole. *The Mathematical Analysis of Logic*. Henderson & Spalding, 1847.

[2] Augustus De Morgan. *Formal Logic or The Calculus of Inference*. Taylor and Walton, 1847.