Homework

1. Prove in Gentzen-style natural deduction:

   (a) $(\forall y.Qy \to Py) \to Qa \to Pa$

   (b) $(\forall x.\forall y.Px \to Py) \to Pa \to \forall x.Px$

   (c) $(\forall x.Px \to A) \to (A \to \forall x.\forall y.Qyx) \to Pa \to Qbb$

   and give λ_P derivations whose types are the conclusions of the above proofs.

   (d) What is the isomorphism between λ_P and predicate logic precisely?
   (Hint: there is supposed to be exactly one inhabited type for each provable proposition)

2. Define in HOL Light the type `drei` that has only 3 elements

   (a) Using the datatype package, as in `ind_types.ml`

   (b) Using the kernel typedef mechanism starting from a known bigger type (for example `num`)

   (c) Prove in HOL Light that $\exists x, y, z \in \texttt{drei}. x \neq y \neq z \neq x$

3. Real numbers in HOL light

   - Find the HOL Light type definition of real numbers (including the types it is created from)
   - Try to define the real numbers yourself, as a quotient of converging Cauchy sequences
   - Which basic properties can be easily define? (zero? one? addition? division?)