Homework

1. Define in HOL Light the type of lambda terms

   (a) Define the inductive type lam_0 that represents the lambda terms

   (b) Define the function f_0 that computes the free variables of a lambda term ($f : lam_0 \to set$)

   (c) Define α-equivalence relation on the raw lambda terms (the type lam_0)

   (d) Define the type lam as a quotient of lam_0 by the α-equivalence relation

   (e) Lift the constructors of lam_0 and the free variables function f_0 to the quotient type

   (f) Prove the equations that relate the lifted constructors to the lifted free variable function f

   (g) ($\star$) Define substitution on the quotient type

   (some hints added above)

2. For each of the following λ_2 find a term, or give a model that would say why this is not possible:

   (a) $(\forall b.\, b) \to a$

   (b) $\forall a.\, \forall b.\, ((a \to b) \lor (b \to a))$

   (c) $\forall a.\, (a \lor \neg a)$

   (d) $\forall a.(a \to \forall b.(b \to (a \land b)))$

   (e) $\forall b.(\forall a.((a \to b) \land (b \to a)))$

   (f) ($\star$) $\exists a.\exists b.((a \lor b) \land (\neg a \lor \neg b))$

   Hint: We have not formally defined what a "model" in λ_2 is, so presenting such a notion is a good way to start the exercise for the non-provable theorems.