## Third Exam

## Logic Programming, LVA 703113

October 2, 2015

The exam consists of 6 exercises with a total of 100 points. Please fill out your name and credentials before you start the exam.


| $0-49:$ | $\mathbf{5}$ | $50-59:$ | $\mathbf{4}$ | $60-74:$ | $\mathbf{3}$ | $75-89:$ | $\mathbf{2}$ | $90-100:$ | $\mathbf{1}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |

1. Consider the directed graph $G=(\{a, b, c, d, e, f, g\}, E)$ with the following set of edges:

$$
E=\{(a, b),(a, c),(b, d),(c, d),(d, e),(f, g)\}
$$

- Represent $G$ in Prolog and implement a relation connected/2 that expresses that two nodes are connected in $G$.
- Show that for any graph $G$ the size of the search tree for a ground query is bounded quadratic in the number of vertices of $G$.
- Is the above estimate on the size of the search tree also true for non-ground queries?

2. Implement a predicate duplicate/3 that duplicates the elements of a list a given number of times. For example the query duplicate ( $[\mathrm{a}, \mathrm{b}, \mathrm{c}], 2, \mathrm{Xs}$ ) should deliver the answer $\mathrm{Xs}=[\mathrm{a}, \mathrm{a}, \mathrm{b}, \mathrm{b}, \mathrm{c}, \mathrm{c}]$. Use difference-lists in your implementation, where you can assume that $\backslash$ seperates difference lists.
3. Implement a predicate isotree (Tree ${ }_{1}$, Tree $_{2}$ ) which holds if Tree $_{1}$, Tree $_{2}$ are isomorphic binary trees.
Hint: You can use any suitable representation of binary trees.
4. Consider the following grammar for propositional formulas over the atoms $\mathrm{p}, \mathrm{q}$, and $r$ :

$$
\begin{array}{ll}
P \rightarrow \text { true } \mid \text { false } & P \rightarrow \neg P \\
P \rightarrow(P \wedge P) & P \rightarrow(P \vee P)
\end{array}
$$

- Write a DCG that generates the languages by directly encoding the grammar and builds an expression tree for the formula parsed.
- Improve your implementation by taking into account the following precedence of connectives $\neg>\wedge>\vee$, so that brackets can be dropped. Furthermore prevent the left-recursion in the grammar.

5. Implement (part of) the Knight's tour problem: how can a knight jump on an $N \times N$ chessboard in such a way that it visits every square exactly once?
Hint: Represent the squares by pairs of their coordinates of the form $X / Y$, where $X$ and $Y$ are integers between 1 and $N$. It suffices to implement the relation jump $(\mathrm{N}, \mathrm{X} / \mathrm{Y}, \mathrm{U} / \mathrm{V})$ to express the fact that a knight can jump from $X / Y$ to $U / V$ on a $N \times N$ chessboard.
6. Determine whether the following statements are true or false. Every correct answer is worth 2 points, every wrong answer -1 points. (The worst that can happen is that you get zero points for this exercise.)
statement
yes
no

A rule is a universally quantified logical formula of the form $A \leftarrow B_{1}, B_{2}, \ldots, B_{n}$, $\square$
$\square$ where $A$ is a goal and for all $i=1, \ldots, n$ : $B_{i}$ is a goal.

An SLD-refutation is a finite SLD-derivation ending in the goal to be proven.


Logic programming is a declarative programming paradigm, that is, the computation of a function is made a first-class citizen.

The declarative semantics of a program $P$ is the minimal model of $P$.

The order of goals is irrelevant in the computation model of logic programming, but not the order of rules.

A search tree is the same as an SLD tree.

Prolog is a language without types and the main technique to manipulate data is unification.

Difference lists are ineffective if the generation of different sections of a list depend on each other.

A meta-interpreter in Prolog interprets Prolog terms on the Warren abstract machine.

The predicate bagof (Template,Goal,Bag) unifies Bag with the alternatives of
 Goal that meet Template.

