Logic Programming

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Summary of Last Lecture

Observation

the basic operations of relational algebras, namely:

1. union
2. difference
3. cartesian product
4. projection
5. selection
6. intersection

can easily be expressed within logic programming

Definition

- a type is a (possible infinite) set of terms
- types are conveniently defined by unary relations

Outline of the Lecture

Logic Programs

introduction, basic constructs, database and recursive programming, theory of logic programs

The Prolog Language

programming in pure Prolog, arithmetic, structure inspection, meta-logical predicates, cuts, extra-logical predicates, how to program efficiently

Advanced Prolog Programming Techniques

nondeterministic programming, incomplete data structures, definite clause grammars, meta-programming, constraint logic programming

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  consists of sequence $B_1, \dots, B_n$ of goals
- empty goal clause $\leftarrow$ is denoted by $\square$
- resolvent of goal clause $\leftarrow B_1, \dots, B_i, \dots, B_m$ and rule $A \leftarrow A_1, \dots, A_n$
  is goal clause

$$\leftarrow B_1\theta, \dots, B_{i-1}\theta, A_1\theta, \dots, A_n\theta, B_{i+1}\theta \dots, B_m\theta$$

  provided B_i (selected goal) and A unify with mgu θ

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  - C_i has no variables in common with $G, C_0, \ldots, C_{i-1}$
- SLD-refutation is finite SLD-derivation ending in $\square$
- computed answer substitution of SLD-refutation of P and G with substitutions $\theta_0, \theta_1, \ldots, \theta_m$ is restriction of $\theta_0\theta_1 \cdots \theta_m$ to variables in G

Example

```
plus(0,X,X).
plus(s(X),Y,s(Z)) ← plus(X,Y,Z).
times(0,X,0).
times(s(X),Y,Z) ← times(X,Y,U), plus(U,Y,Z).

← times(X,X,Y)
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G_2: $\leftarrow$ plus(0,s(0),Y)

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G_3: $\square$

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G_3: $\square$ computed answer substitution: $\mathtt{X} \mapsto \mathtt{s(0)},\ \mathtt{Y} \mapsto \mathtt{s(0)}$

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selected goal in goal clause is determined by selection function

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2. rule in logic program

substitution – avoid choice by always taking mgu

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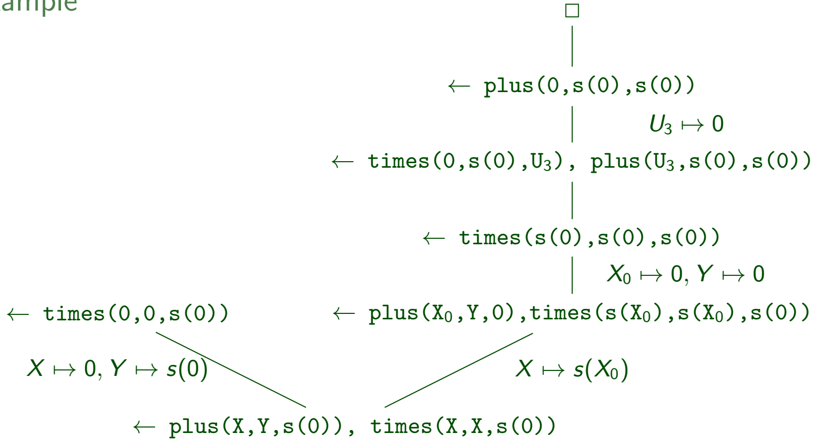

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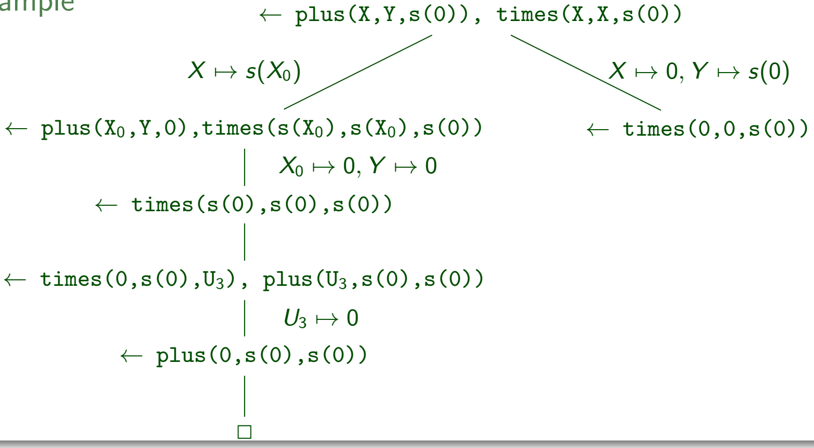

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- the minimal model of P is the intersection of all models; the minimal model is unique

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Definition

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the denotational semantics assign meanings to programs based on associating with the program a function over the domain computed by the program

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a program P is called

- correct with respect to the intended meaning M, if the meaning of P is a subset of M
- complete if the intended meaning M is a subset of the meaning of P

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3. case distinction on $N = 0$ and $N > 0$

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6. $t = s^{m'}(0)$ for some $m' \in \mathbb{N}$
7. $\texttt{natural\_number}(s^{m'1}(0)) \in M$ and $m = m' + 1$

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is the program is correct wrt the following set?

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7. $\texttt{natural\_number}(s^m(0)) \in M$ iff $m \leqslant K$
8. what happens for $m > K$?

Example

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natural_number(0).
natural_number(s(X)) ← natural_number(X).

plus(0,X,X) ← natural_number(X).
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Proof Sketch.

1. completeness: suppose $X + Y = Z$; then we give an SLD tree of $\texttt{plus}(X, Y, Z)$
2. correctness: suppose $\texttt{plus}(X, Y, Z)$ is deducible; then we prove by induction on the length of this deduction that $X + Y = Z$

∎

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Remark

a proof tree is a different representation of one successful solution represented by a search tree