# First Exam
# Automated Reasoning, LVA 703608

July 4, 2017

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**Name**: **Study Program Number**:

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The exam consists of 3 exercises with a total of 60 points. Please fill out your name and credentials *before* you start the exam.

| 1 | 2 | 3 | Sum |
|---|---|---|---|
| ☐ | ☐ | ☐ | ☐ |

1. Consider *free-variable* semantic tableaux.
   a) Define the $\gamma$- and $\delta$-rule. (5 pts)
   b) Prove soundness of the $\delta$ rule. (15 pts)

2. Prove the following lifting lemma for paramodulation.

   **Lemma.** Let $\tau_1$ and $\tau_2$ be a ground substitution and consider the inference:

$$\frac{C\tau_1 \vee (s = t)\tau_1 \quad D\tau_2 \vee L\tau_2[x\tau_2]}{C\tau_1 \vee D\tau_2 \vee L\tau_2[f(t\tau_1)]} \ ,$$

   where $x\tau_2 = f(\ldots, s'\tau_3, \ldots)$ and $s\tau_1 = s'\tau_3$. Here $f$ is function symbol of suitable arity. Then $C \vee D \vee L[f(\ldots, t, \ldots)]$ is derivable from (i) $C \vee s = t$ and (ii) $D \vee L[x]$ making (iii) use of (a) suitable functional reflexivity equation(s). (20 pts)

3. Determine whether the following statements are true or false. Every correct answer is worth 2 points, every wrong answer -1 points. (The worst that can happen is that you get zero points for this exercise.) (20 pts)

| **Statement** | **yes** | **no** |
|---|---|---|
| Let $\mathcal{G}$ be a set of universal sentences (of $\mathcal{L}$) without $=$. Then $\mathcal{G}$ is satisfiable iff $\mathcal{G}$ has a Herbrand model (over $\mathcal{L}$). | ☐ | ☐ |
| There exists exactly one path in a semantic tree that gives rise to a (partial) Herbrand interpretations. | ☐ | ☐ |
| A tableau proof for $F$ is a closed tableau for $\{F\}$. | ☐ | ☐ |
| A strategy $S$ is fair if for any sequence of tableaux $T_1, T_2, \ldots$ following $S$ we have for each $i \in \mathbb{N}$: (i) Every non-literal formula in $T_i$ is eventually expanded on each branch it occurs, and (ii) every $\delta$-formula occurrence in $T_i$ has the $\delta$-rule applied to it arbitrarily often on each branch it occurs. | ☐ | ☐ |
| The Herbrand complexity of an unsatisfiable clause set $\mathcal{C}$ is the cardinality of the smallest subset of ground instances of $\mathcal{C}$ which is unsatisfiable. | ☐ | ☐ |
| For an inner Skolemisation step the arguments of the introduced Skolem function are a subset of the free variables in the scope of the existentially quantified variable replaced. | ☐ | ☐ |
| The antiprenex form of an NNF $A$ is obtained my maximising the quantifier range by quantifier shifting rules. | ☐ | ☐ |
| Suppose literal $L$ is strictly larger than any other literal in a clause $C$ wrt. some proper literal order $\succ_{\mathsf{L}}$. Then $L$ is also strictly maximal wrt. $C$. | ☐ | ☐ |
| Superposition with equations is sound, but not (refutationally) complete. | ☐ | ☐ |
| We say a ground clause set is saturated upto redundancy if all inferences from non-redundant premises are redundant. | ☐ | ☐ |