



- [3] 1 Draw the equality graphs for the following equality logic conjunctions, identify all simple cycles, and decide whether the formulas are satisfiable.

$$(a) \quad x_1 = x_3 \wedge x_1 \neq x_4 \wedge x_3 \neq x_7 \wedge x_4 \neq x_6 \wedge x_6 \neq x_7 \wedge x_5 = x_9 \wedge \\ x_5 = x_7 \wedge x_8 \neq x_9 \wedge x_9 = x_{10} \wedge x_7 = x_9 \wedge x_5 \neq x_8$$

$$(b) \quad x_1 = x_2 \wedge x_2 = x_3 \wedge x_3 = x_4 \wedge x_4 \neq x_6 \wedge x_1 \neq x_5 \wedge x_6 \neq x_1 \wedge x_4 = x_9 \wedge \\ x_4 = x_1 \wedge x_1 \neq x_9 \wedge x_6 = x_7 \wedge x_7 = x_8 \wedge x_5 \neq x_8 \wedge x_2 = x_9 \wedge x_4 = x_8$$

- 2 Consider the following system of linear inequalities (cf. slide 14 of week 8):

$$\begin{aligned} -4x - 5y &\leq -10 \\ 2x - 5y &\leq 0 \\ -x + 5y &\leq 21 \\ -7x + y &\leq -4 \\ 5x + 2y &\leq 25 \end{aligned}$$

- [3] (a) Use (an implementation of) the DPLL(T) Simplex algorithm to find a solution to the problem in \mathbb{R}^2 . Derive a Gomory cut from the solution, and draw it in the diagram.

- [2] \star (b) Run DPLL(T) Simplex on this modified problem, and repeat the procedure until you found a solution in \mathbb{Z}^2 .

- [4] 3 The schedule for the IT helpdesk staff of the Loanly Officers banking group is currently set up in a manual process using excel sheets; however, there are scenarios where the generated schedule violates some constraints. In order to deal with this issue, scheduling should be automated. The constraints are as follows:

- (a) There are three types of 8-hour shifts: morning shift (06:00–14:00), afternoon shift (14:00–22:00), night shift (22:00–06:00).
- (b) The staff consists of 7 persons who work morning, afternoon, and night in 8-hour shifts to provide a service that operates 24 hours a day and 7 days a week.
- (c) The morning and afternoon shifts should include two employees.
- (d) The night shift is done by one person.
- (e) Each employee is required to work 40 hours per week.
- (f) Employees must not work on day shift after a night shift.
- (g) Each employee must have at most one shift per day.
- (h) Joe and Ben don't do night shifts.
- (i) Sally does not work on weekends.
- (j) Andy and Chris should not work together since they always fight.

Can you find a week schedule for the staff, using a linear integer arithmetic encoding?