Exercises 10 June 14, 2018

[6] 1 Use the Nelson-Oppen procedure to determine satisfiability of the following formulas:

(a) The following formula combines uninterpreted functions and linear real arithmetic:

$$z = 0 \wedge y \leq x \wedge x \leq y + z \wedge \mathsf{f}(y) = \mathsf{f}(z) \wedge \mathsf{f}(y) = 1 \wedge \mathsf{f}(z) = 2$$

(b) The following formula combines uninterpreted functions and linear real arithmetic:

$$x = y + 1 \wedge y \leq z \wedge x \geq z + 1 \wedge \mathsf{f}(y) = \mathsf{a} \wedge \mathsf{f}(z) = \mathsf{b}$$

(c) The following formula combines uninterpreted functions and linear *integer* arithmetic:

$$1 \leq x \wedge x \leq 2 \wedge \mathsf{f}(1) = \mathsf{a} \wedge \mathsf{f}(x) = \mathsf{b} \wedge \mathsf{a} = \mathsf{b} + 2 \wedge \mathsf{f}(2) = \mathsf{f}(1) + 3$$

[2] ⋆ 2 Is the theory of bit vectors convex? Give a proof or a counterexample.

[4] 3 Describe how you can use an SMT encoding to solve a real-world problem you (or your friends or acquaintances) had to solve at some point. It can be related to some topic discussed in a university course, work, or leisure activities, but it should be different from the SMT applications mentioned in this course.

You do not have to give an actual encoding, but describe in detail how it could be done, including:

- which logic is convenient to use (pure SAT, arithmetic, uninterpreted functions, bitvectors, a combination, ...),
- which variables are needed for which purpose,
- which type of constraints are needed, and how they can be formulated.

(To exclude applications like finding the solution to $1 + 2$, let's say that at least 10 variables should be required.)