SAT and SMT Solving

Sarah Winkler

SS 2018

Department of Computer Science
University of Innsbruck

Outline

- Summary of Last Week
- Conflict Analysis
- Conflict Driven Clause Learning
- Bit Vectors

Summary of Last Week

Approach

- most state-of-the-art SAT solvers use variation of Davis - Putnam - Logemann - Loveland (DPLL) procedure (1962)
- DPLL is sound and complete backtracking-based search algorithm
- can be described abstractly by transition system (Nieuwenhuis, Oliveras, Tinelli 2006)

Definition (Abstract DPLL)

- decision literal is annotated literal l^d
- state is pair $M \parallel F$ for
  - list M of (decision) literals
  - formula F in CNF
- transition rules

$$M \parallel F \qquad \Longrightarrow \qquad M' \parallel F' \quad \text{or} \quad \text{FailState}$$

Definition (DPLL Transition Rules)

- unit propagation $\qquad M \parallel F,\ C \lor l \quad \Longrightarrow \quad M\ l \parallel F,\ C \lor l$
  if $M \vDash \neg C$ and l is undefined in M
- pure literal $\qquad M \parallel F \quad \Longrightarrow \quad M\ l \parallel F$
  if l occurs in F but l^c does not occur in F, and l is undefined in M
- decide $\qquad M \parallel F \quad \Longrightarrow \quad M\ l^d \parallel F$
  if l or l^c occurs in F, and l is undefined in M
- backtrack $\qquad M\ l^d\ N \parallel F, C \quad \Longrightarrow \quad M\ l^c \parallel F, C$
  if $M\ l^d\ N \vDash \neg C$ and N contains no decision literals
- fail $\qquad M \parallel F, C \quad \Longrightarrow \quad$ FailState
  if $M \vDash \neg C$ and M contains no decision literals
- backjump $\qquad M\ l^d\ N \parallel F, C \quad \Longrightarrow \quad M\ l' \parallel F, C$
  if $M\ l^d\ N \vDash \neg C$ and $\exists$ clause $C' \lor l'$ such that
  - $F, C \vDash C' \lor l'$ $\qquad$ backjump clause
  - $M \vDash \neg C'$ and l' is undefined in M, and l' or l'^c occurs in F or in $M\ l^d\ N$

Definition

basic DPLL $\mathcal{B}$ consists of unit propagation, decide, fail, and backjump

Theorem (Termination)

there are no infinite derivations $\quad \| \, F \quad \Longrightarrow_{\mathcal{B}} \quad S_1 \quad \Longrightarrow_{\mathcal{B}} \quad S_2 \quad \Longrightarrow_{\mathcal{B}} \quad \dots$

Theorem (Correctness)

for derivation with final state S_n:

$$\| \, F \quad \Longrightarrow_{\mathcal{B}} \quad S_1 \quad \Longrightarrow_{\mathcal{B}} \quad S_2 \quad \Longrightarrow_{\mathcal{B}} \quad \dots \quad \Longrightarrow_{\mathcal{B}} \quad S_n$$

- *if* $S_n = \text{FailState}$ *then* F *is unsatisfiable*
- *if* $S_n = M \| F'$ *then* F *is satisfiable and* $M \vDash F$

Conflict Analysis

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Backjump: Idea

- backjump clause $C' \vee l'$ is entailed by formula (magically detected)
- prefix M of current literal list entails $\neg C'$, so l' must be true

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Backjump to Definition

- backjump $\quad M\, l^d\, N \parallel F, C \quad \Longrightarrow \quad M\, l' \parallel F, C$

  if $M\, l^d\, N \models \neg C$ and $\exists$ clause $C' \vee l'$ such that
  - $F, C \models C' \vee l'$ backjump clause
  - $M \models \neg C'$ and l' is undefined in M, and l' or l'^c occurs in F or in $M\, l^d\, N$

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  - $F, C \vDash C' \vee l'$ — backjump clause
  - $M \vDash \neg C'$ and l' is undefined in M, and l' or l'^c occurs in F or in $M\, l^d\, N$

Example

$1^d\ 2\ 3^d\ 4^d\ \overline{5} \parallel \overline{1} \vee 2,\ \overline{1} \vee \overline{3} \vee 4 \vee 5,\ \overline{2} \vee \overline{4} \vee \overline{5},\ 4 \vee \overline{5},\ \overline{4} \vee 5,\ \overline{1} \vee \overline{5} \vee 6,\ \overline{2} \vee \overline{5} \vee \overline{6}$

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$M = 1^d\, 2 \qquad l = 3 \qquad N = 4^d\, \overline{5} \qquad C = \overline{4} \vee 5 \qquad C' = \overline{1} \qquad l' = \overline{5}$

- $1^d\, 2\, 3^d\, 4^d\, \overline{5} \vDash \neg(\overline{4} \vee 5)$

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Backjump to Definition

- backjump $\quad M\ l^d\ N \parallel F, C \implies M\ l' \parallel F, C$
  if $M\ l^d\ N \vDash \neg C$ and $\exists$ clause $C' \vee l'$ such that
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- $1^d\; 2 \vDash 1$

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Backjump to Definition

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$$\Longrightarrow\ 1^d\ 2\ \overline{5} \parallel \overline{1} \vee 2,\ \overline{1} \vee \overline{3} \vee 4 \vee 5,\ \overline{2} \vee \overline{4} \vee \overline{5},\ 4 \vee \overline{5},\ \overline{4} \vee 5,\ \overline{1} \vee \overline{5} \vee 6,\ \overline{2} \vee \overline{5} \vee \overline{6}$$

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Desirable Properties of Backjump Clauses

- small
- should trigger progress

How to Determine Backjump Clauses?

- implication graph
- resolution

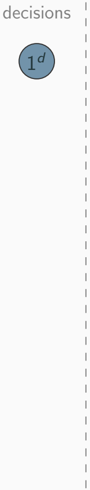
Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
$$(\overline{7} \vee 8 \vee \overline{9} \vee 10) \wedge (\overline{10} \vee \overline{11}) \wedge (\overline{10} \vee 12) \wedge (\overline{12} \vee \overline{13}) \wedge (6 \vee 11 \vee 13)$$

decisions

Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
$$(\overline{7} \vee 8 \vee \overline{9} \vee 10) \wedge (\overline{10} \vee \overline{11}) \wedge (\overline{10} \vee 12) \wedge (\overline{12} \vee \overline{13}) \wedge (6 \vee 11 \vee 13)$$



level	literal	reason
1	1	decision

Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
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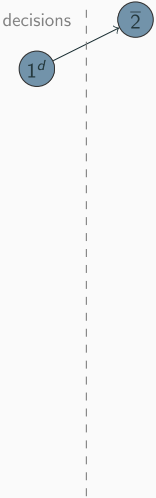

level	literal	reason
1	1	decision
	$\overline{2}$	$\overline{1} \vee \overline{2}$

Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
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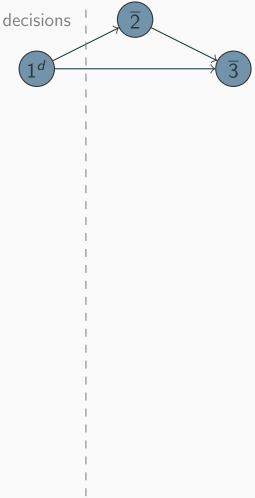

level	literal	reason
1	1	decision
	$\overline{2}$	$\overline{1} \vee \overline{2}$
	$\overline{3}$	$\overline{1} \vee 2 \vee \overline{3}$

Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
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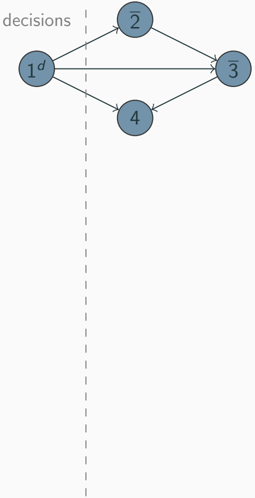

level	literal	reason
1	1	decision
	$\overline{2}$	$\overline{1} \vee \overline{2}$
	$\overline{3}$	$\overline{1} \vee 2 \vee \overline{3}$
	4	$\overline{1} \vee 3 \vee 4$

Example: Implication Graph

$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$
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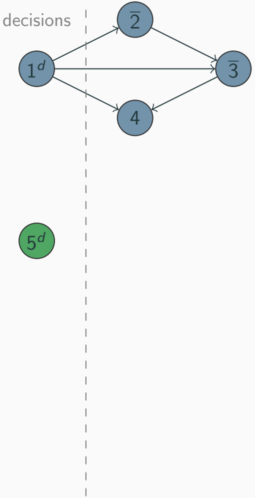

level	literal	reason
1	1	decision
	$\overline{2}$	$\overline{1} \vee \overline{2}$
	$\overline{3}$	$\overline{1} \vee 2 \vee \overline{3}$
	4	$\overline{1} \vee 3 \vee 4$
2	5	decision

Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
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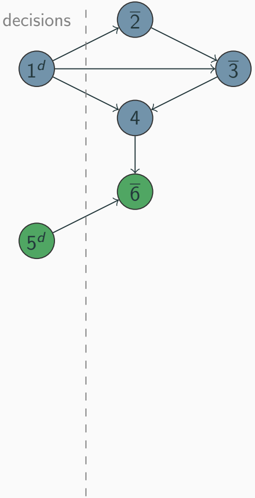

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Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
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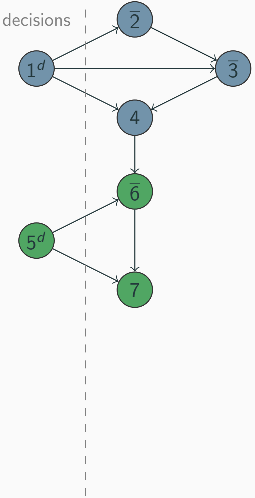
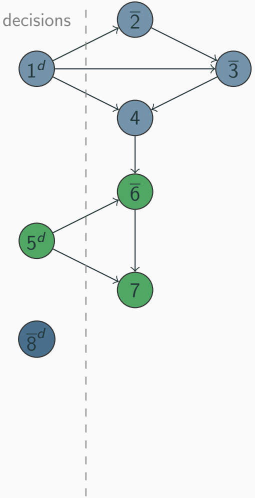


level	literal	reason
1	1	decision
	$\overline{2}$	$\overline{1} \vee \overline{2}$
	$\overline{3}$	$\overline{1} \vee 2 \vee \overline{3}$
	4	$\overline{1} \vee 3 \vee 4$
2	5	decision
	$\overline{6}$	$\overline{4} \vee \overline{5} \vee \overline{6}$
	7	$\overline{5} \vee 6 \vee 7$

Example: Implication Graph

$$\varphi = (\overline{1} \lor \overline{2}) \land (\overline{1} \lor 2 \lor \overline{3}) \land (\overline{1} \lor 3 \lor 4) \land (\overline{4} \lor \overline{5} \lor \overline{6}) \land (\overline{5} \lor 6 \lor 7) \land$$
$$(\overline{7} \lor 8 \lor \overline{9} \lor 10) \land (\overline{10} \lor \overline{11}) \land (\overline{10} \lor 12) \land (\overline{12} \lor \overline{13}) \land (6 \lor 11 \lor 13)$$



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1	1	decision
	$\overline{2}$	$\overline{1} \lor \overline{2}$
	$\overline{3}$	$\overline{1} \lor 2 \lor \overline{3}$
	4	$\overline{1} \lor 3 \lor 4$
2	5	decision
	$\overline{6}$	$\overline{4} \lor \overline{5} \lor \overline{6}$
	7	$\overline{5} \lor 6 \lor 7$
3	$\overline{8}$	decision

Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
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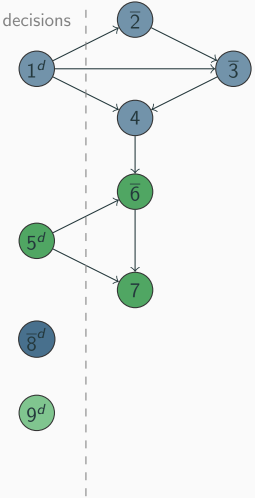

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2	5	decision
	$\overline{6}$	$\overline{4} \vee \overline{5} \vee \overline{6}$
	7	$\overline{5} \vee 6 \vee 7$
3	$\overline{8}$	decision
4	9	decision

Example: Implication Graph

$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$
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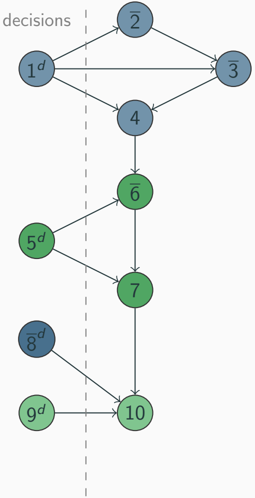

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	4	$\overline{1} \vee 3 \vee 4$
2	5	decision
	$\overline{6}$	$\overline{4} \vee \overline{5} \vee \overline{6}$
	7	$\overline{5} \vee 6 \vee 7$
3	$\overline{8}$	decision
4	9	decision
	10	$\overline{7} \vee 8 \vee \overline{9} \vee 10$

Example: Implication Graph

$$\varphi = (\overline{1} \vee \overline{2}) \wedge (\overline{1} \vee 2 \vee \overline{3}) \wedge (\overline{1} \vee 3 \vee 4) \wedge (\overline{4} \vee \overline{5} \vee \overline{6}) \wedge (\overline{5} \vee 6 \vee 7) \wedge$$
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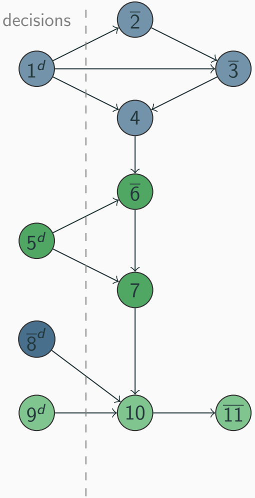

level	literal	reason
1	1	decision
	$\overline{2}$	$\overline{1} \vee \overline{2}$
	$\overline{3}$	$\overline{1} \vee 2 \vee \overline{3}$
	4	$\overline{1} \vee 3 \vee 4$
2	5	decision
	$\overline{6}$	$\overline{4} \vee \overline{5} \vee \overline{6}$
	7	$\overline{5} \vee 6 \vee 7$
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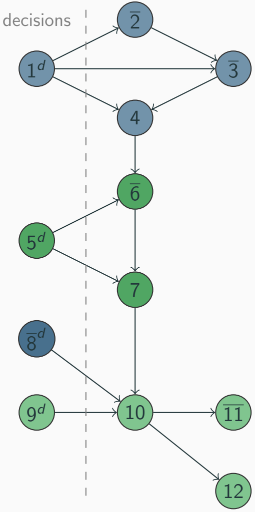

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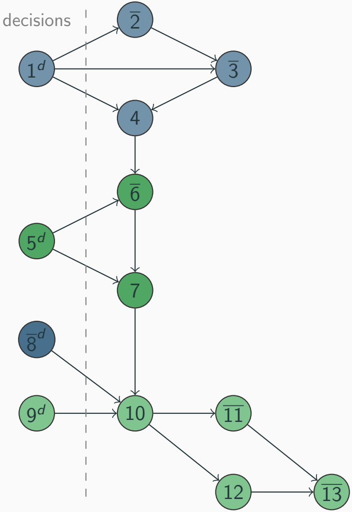

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	$\overline{2}$	$\overline{1} \lor \overline{2}$
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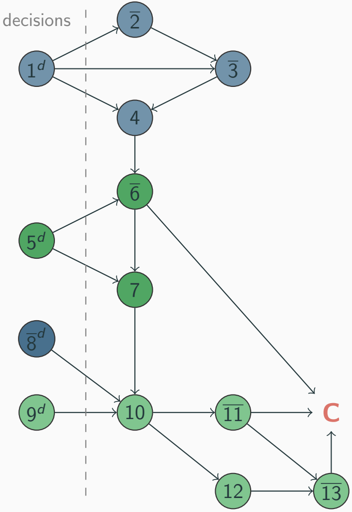

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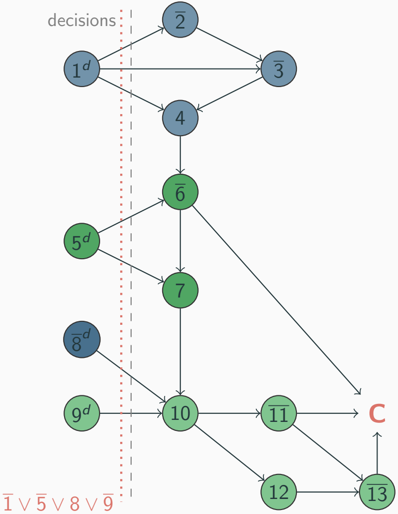

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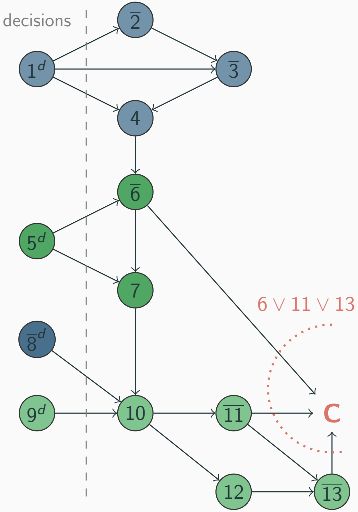

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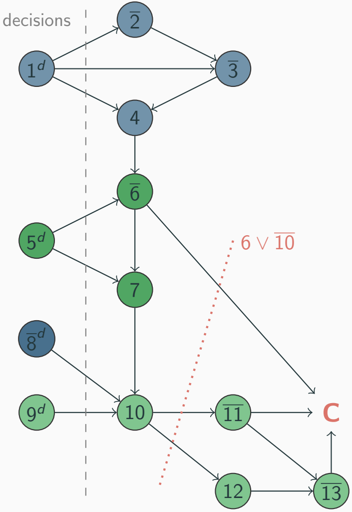

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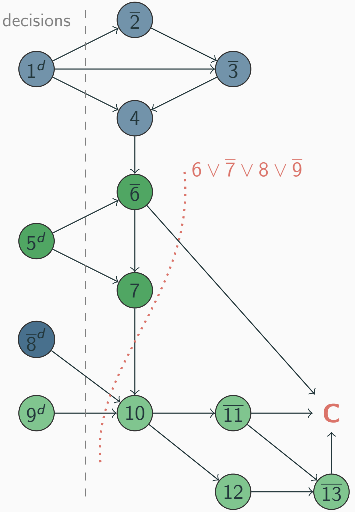

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potential backjump clause

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or former backjump clause

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Conflict Driven Clause Learning

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- *if* $S_n = M \| F'$ *then* F *is satisfiable and* $M \models F$

Bit Vectors

Outline

- Summary of Last Week
- Conflict Analysis
- Conflict Driven Clause Learning
- Bit Vectors

Application: Collision Attacks using SAT

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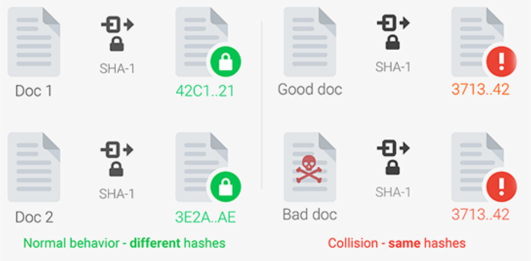

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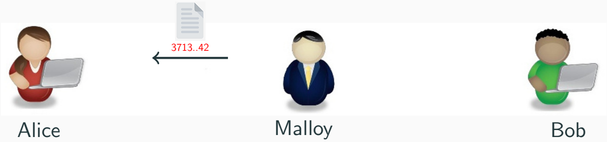

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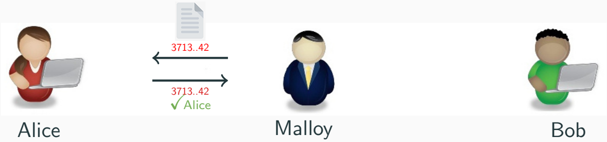

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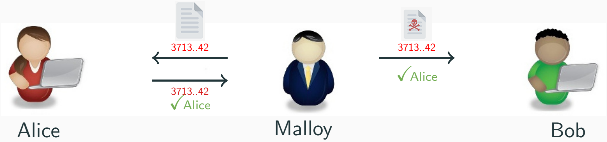

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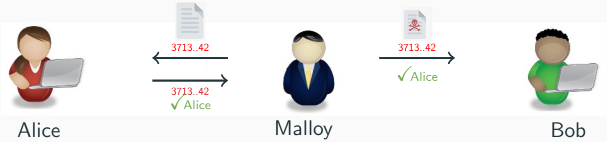

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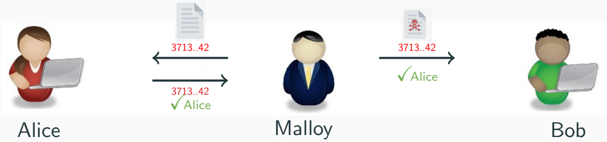

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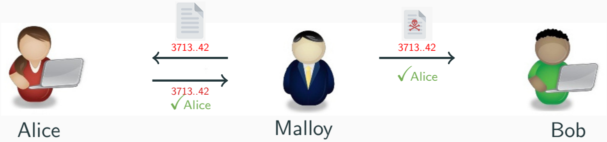

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More Cryptanalysis using SAT

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Tools for SAT/SMT-Based Cryptanalysis

- CryptoMiniSat
- CryptoSMT
- Transalg
- ...

Bit Blasting

Representing Characters as Bit Vectors

- ASCII character a is representable as bit vector for `char` of 8 bits:
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Example (Rotation Hash)

```
def rotation_hash(s):
  h = 0
  for i in range(0,len(s)):
    h = (h << 4) ^ (h >> 28) ^ ord(s[i])
  return h
```

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common open source SAT/SMT solver

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- `Xor(`a, b`)` exclusive or

Solving Formulas

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Moreover ...

- `simplify(`φ`)` simplifies formula φ
- `Solver.statistics()` is map of solving statistics

Example

```
from z3 import *
p = Bool('p') # create variable named 'p'
foo1 = FreshBool('foo') # create new variables prefixed 'foo'
foo2 = FreshBool('foo')

phi = Or(p, p, And(foo2, Xor(foo1, Not(foo1)), True), False)
print(phi) # Or(p, p, And(foo!1, Xor(foo!0, Not(foo!0)), True), False)
psi = simplify(phi)
print(psi) # Or(p, foo!1)

solver = Solver()
solver.add(psi) # assert that psi should be true
solver.add(Implies(foo1,p), Or(foo1, foo2)) # assert something else

print solver # [Or(p, foo!1), Implies(foo!0, p), Or(foo!0, foo!1)]
result = solver.check() # check for satisfiability

if result:
  model = solver.model() # get valuation
  print model[p], model[foo1], model[foo2] # False False True
```