SAT and SMT Solving

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Outline

Summary of Last Week

Definitions

- theory consists of
  - signature Σ: set of function and predicate symbols
  - axioms T: set of sentences in first-order logic in which only function and predicate symbols of Σ appear
- theory is stably infinite if every satisfiable quantifier-free formula has model with infinite carrier set
- theory T is convex if $F \vDash_T \bigvee_{i=1}^{n} u_i = v_i$ implies $F \vDash_T u_i = v_i$ for some $1 \leqslant i \leqslant n$ $\forall$ quantifier-free conjunction F and variables u_i, v_i

Definition

theory combination $T_1 \oplus T_2$ of two theories

- T_1 over signature Σ_1
- T_2 over signature Σ_2

has signature $\Sigma_1 \cup \Sigma_2$ and axioms $T_1 \cup T_2$

Assumptions

Nelson-Oppen Method: Nondeterministic Version

Input quantifier-free conjunction φ in theory combination $T_1 \oplus T_2$

Output satisfiable or unsatisfiable

1. purification

$$\varphi \;\approx\; \varphi_1 \wedge \varphi_2 \quad \text{for } \Sigma_1\text{-formula } \varphi_1 \text{ and } \Sigma_2\text{-formula } \varphi_2$$

2. guess and check
   - V is set of shared variables in φ_1 and φ_2
   - guess equivalence relation E on V
   - arrangement $\alpha(V, E)$ is formula

$$\bigwedge_{x \mathrel{E} y} x = y \quad \wedge \bigwedge_{\neg(x \mathrel{E} y)} x \neq y$$

   - if $\varphi_1 \wedge \alpha(V, E)$ is T_1-satisfiable and $\varphi_2 \wedge \alpha(V, E)$ is T_2-satisfiable then return satisfiable else return unsatisfiable

Nelson-Oppen Method: Deterministic Version

Input quantifier-free conjunction φ in combination $T_1 \oplus T_2$ of convex theories T_1 and T_2

Output satisfiable or unsatisfiable

1. purification $\varphi \;\approx\; \varphi_1 \wedge \varphi_2$ for Σ_1-formula φ_1 and Σ_2-formula φ_2
2. V: set of shared variables in φ_1 and φ_2
   E: already discovered equalities between variables in V
3. test satisfiability of $\varphi_1 \wedge E$ (and add implied equations)
   - if $\varphi_1 \wedge E$ is T_1-unsatisfiable then return unsatisfiable
   - else add new implied equalities to E
4. test satisfiability of $\varphi_2 \wedge E$ (and add implied equations)
   - if $\varphi_2 \wedge E$ is T_2-unsatisfiable then return unsatisfiable
   - else add new implied equalities to E
5. if E has been extended in steps 3 or 4 then go to step 2
   else return satisfiable

Outline

- Summary of Last Week
- Bounded Model Checking for Verification

Bounded Model Checking for Verification

Disastrous Software Bugs

Ariane 5 Flight 501 (1996)

- destroyed 37 seconds after launch
- software for Ariane 4 for was reused
- software error: data conversion from 64-bit floating point to 16-bit integer caused arithmetic overflow
- cost: 370 million $

http://en.wikipedia.org/wiki/Ariane_5_Flight_501

Mars Exploration Rover “Spirit” (2004)

- landed on January 4
- stopped communicating on January 21
- software error: stuck in reboot loop
- reboot failed because of flash memory failure, ultimate problem: too many files

http://en.wikipedia.org/wiki/Spirit_(rover)

Heathrow Terminal 5 Opening (2008)

- baggage system collapsed on opening day
- 42,000 bags not shipped with their owners, 500 flights cancelled
- software was tested but did not work properly with real-world load
- cost 50 million £

http://www.zdnet.com/article/it-failure-at-heathrow-t5-what-really-happened

Trading Glitch at Knight Capital (2012)

- bug in trading software resulted in 45 minutes of uncontrolled buys
- company did 11% of US trading that year
- software was run in invalid configuration
- 440 million $ lost

http://en.wikipedia.org/wiki/Knight_Capital_Group

Death in Self-Driving Car Crash (2018)

- person died in accident with Uber’s self-driving car
- victim was wrongly classified by software as non-obstacle

http://www.siliconrepublic.com/companies/uber-bug-crash

Software is Ubiquitous in Critical Systems

transport, energy, medicine, communication, finance, embedded systems, ...

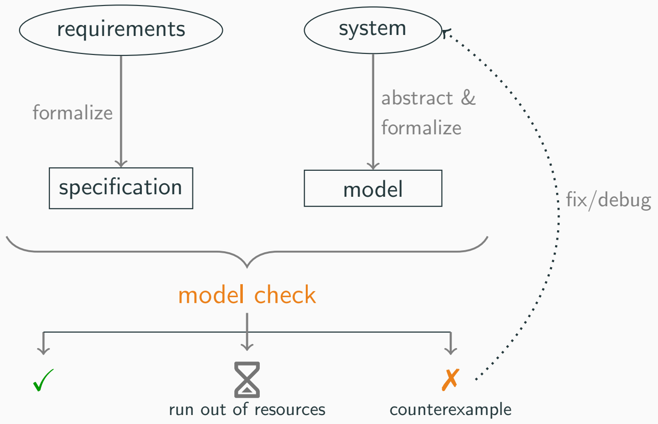
How to Ensure Correctness of Software?

- testing
  - + cheap, simple
  - − checks desired result only for given set of testcases
- verification
  - + can prove automatically that system meets specification, i.e., desired output is delivered for all inputs
  - − more costly

Model Checking

- widely used verification approach to
  - find bugs in software and hardware
  - prove correctness of models
- Turing Award 2007 for Clarke, Emerson, and Sifakis
- bounded model checking can be reduced to SAT/SMT

Model Checking: Workflow

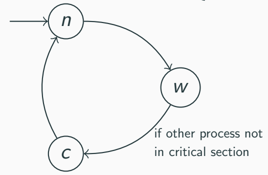


Model Checking Example: Mutex (1)

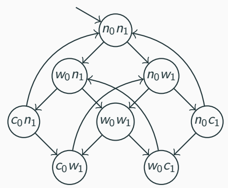
- concurrent processes P_0, P_1 share some resource, access controlled by mutex
- program run by P_0, P_1 matches pattern

```
# non-critical section
while (other process critical) :
  wait ()
# critical section
# non-critical section
```

- process can be abstracted to model $\mathcal{M} = \langle S, R\rangle$ with states $S = \{n, w, c\}$ and transitions R:



Model Checking Example: Mutex (2)



- obtain model for 2 processes by product construction: write $s_0 s_1$ for P_0 being in state s_0 and P_1 in state s_1
- desired properties:

safe: only one process is in its critical section at any time
live: whenever any process wants to enter its critical section, it will eventually be permitted to do so
non-blocking: a process can always request to enter its critical section

- how to formalize desired properties? temporal logic, e.g. LTL or CTL

safe: G $\neg(c_0 \wedge c_1)$ ✓ as $c_0 c_1$ unreachable
live: G $(w_0 \to$ F $c_0)$ ✗ e.g. 
non-blocking: AG $(n_0 \to$ EX $w_0)$ ✓

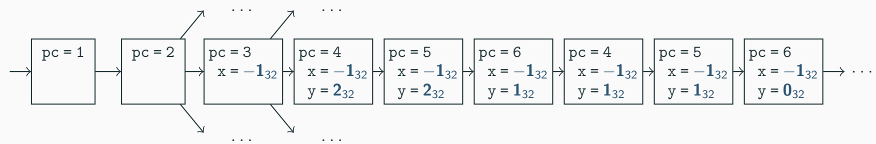
Common Kinds of Properties

- G ψ for propositional formula ψ is safety property
- G $(\psi \to$ F$\chi)$ for propositional formulas ψ,χ is liveness property

Example: Can This Program Cause An Overflow? (1)

```
1 void main() {
2   int x = -1;
3   int y = nondet_int();
4   while (y<100) {
5     y = y+x;
6   }
7 }
```

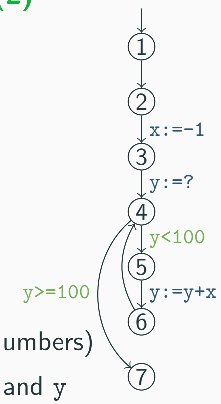
- model checking problem: addition $x + y$ in line 5 does not over/underflow
  - state of program run is assignment of x, y + value of program counter
  - property G $(\mathrm{pc} = 5 \to ((x > \mathbf{0}_{32} \wedge x + y > y) \vee (x \leqslant \mathbf{0}_{32} \wedge x + y \leqslant y)))$
- (part of) model:



- but state space is very large: $(2^{32})^2 \cdot 7$ for bit width 32
- cannot check all possible values

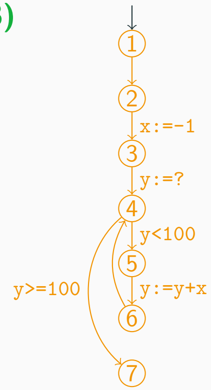
Example: Can This Program Cause An Overflow? (2)

```
1 void main() {
2   int x = -1;
3   int y = nondet_int();
4   while (y<100) {
5     y = y+x;
6   }
7 }
```



- construct program graph G
- $\{1, \dots, 7\}$ are possible values of program counter (line numbers)
- state is tuple $\langle \text{pc}, x, y \rangle$ of values of program counter, x, and y
- state of form $\langle 1, \dots, \dots \rangle$ is initial state
- examples of state transitions according to G:
  - $\langle 4, -\mathbf{1}_{32}, \mathbf{10}_{32} \rangle \to \langle 5, -\mathbf{1}_{32}, \mathbf{10}_{32} \rangle$ is possible
  - $\langle 4, -\mathbf{1}_{32}, \mathbf{101}_{32} \rangle \to \langle 7, -\mathbf{1}_{32}, \mathbf{101}_{32} \rangle$ is possible
  - $\langle 4, \mathbf{10}_{32}, \mathbf{101}_{32} \rangle \to \langle 5, \mathbf{10}_{32}, \mathbf{101}_{32} \rangle$ is not possible
  - $\langle 4, -\mathbf{1}_{32}, \mathbf{1}_{32} \rangle \to \langle 5, -\mathbf{1}_{32}, \mathbf{2}_{32} \rangle$ is not possible

Example: Can This Program Cause An Overflow? (3)



1. define predicates
   - $I(\langle pc, x, y \rangle) = (pc = 1)$ to characterize initial state
   - to characterize possible state transitions:

$$
\begin{aligned}
&T(\langle pc, x, y \rangle, \langle pc', x', y' \rangle) = \\
&\quad (pc = 1 \wedge pc' = 2) \vee (pc = 2 \wedge pc' = 3 \wedge x' = -1) \vee \\
&\quad (pc = 3 \wedge pc' = 4 \wedge x = x') \vee \\
&\quad (pc = 4 \wedge p' = 5 \wedge y < 100 \wedge x = x' \wedge y = y') \vee \\
&\quad (pc = 5 \wedge pc' = 6 \wedge y' = y + x \wedge x = x') \vee \\
&\quad (pc = 4 \wedge pc' = 7 \wedge y \geqslant 100 \wedge x = x' \wedge y = y') \vee \\
&\quad (pc = 6 \wedge pc' = 4 \wedge x = x' \wedge y = y')
\end{aligned}
$$

   - $P(\langle pc, x, y \rangle) = (pc = 5) \wedge ((x > \mathbf{0}_{32} \wedge x + y \leqslant y) \vee (x \leqslant \mathbf{0}_{32} \wedge (y + x > y)))$
2. for states $s_0, \dots, s_k$ formula φ_k expresses overflow occurring within k steps:

$$\varphi_k = I(s_0) \wedge \bigwedge_{i=0}^{k-1} T(s_i, s_{i+1}) \wedge \bigvee_{i=0}^{k} P(s_i)$$

3. if φ_k satisfiable then overflow can occur within k steps ... ask Z3

Bounded Model Checking

- find counterexamples to desired property of transition system (bugs)
- counterexamples are bounded in size

Definition (Transition System)

transition system $\mathcal{T} = (S, \to, S_0, L)$ where

- S is set of states
- $\to \subseteq S \times S$ is transition relation
- $S_0 \subseteq S$ is set of initial states
- A is a set of propositional atoms
- $L : S \to 2^A$ is labeling function associating state with subset of A

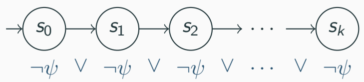
Remark

S and A may be (countably) infinite

Bounded Model Checking: Safety Properties

Idea

given transition system and property G ψ, look for counterexamples in $\leqslant k$ steps



SAT/SMT Encoding

given transition system $\mathcal{T}$ and safety property G ψ

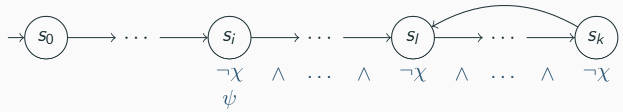
- use encoding $\langle s \rangle$ of state $s \in S$ by set of SAT/SMT variables
- use predicates
  - I for initial states such that $I(\langle s \rangle)$ is true iff $s \in S_0$
  - T for transitions such that $T(\langle s \rangle, \langle s' \rangle)$ is true iff $s \to s'$ in $\mathcal{T}$
  - P such that $P(\langle s \rangle)$ is true iff ψ holds in s
- use different fresh variables for $k + 1$ states $\langle s_0 \rangle, \dots, \langle s_k \rangle$
- check satisfiability of

$$I(\langle s_0 \rangle) \wedge \bigwedge_{i=0}^{k-1} T(\langle s_i \rangle, \langle s_{i+1} \rangle) \wedge \bigvee_{i=0}^{k} \neg P(\langle s_i \rangle)$$

Bounded Model Checking: Liveness Properties

Idea

- counterexample to liveness property $\mathsf{G}\,(\psi \to \mathsf{F}\chi)$ requires infinite path
- look for counterexamples in $\leqslant k$ steps of lasso shape:



SAT/SMT Encoding

given transition system $\mathcal{T}$ and liveness property $\mathsf{G}\,(\psi \to \mathsf{F}\chi)$

- use encoding of states, predicates I and T as for safety properties
- predicate P such that $P(\langle s\rangle)$ is true iff ψ holds in s
- predicate C such that $C(\langle s\rangle)$ is true iff χ holds in s
- check satisfiability of

$$I(\langle s_0\rangle) \wedge \bigwedge_{i=0}^{k-1} T(\langle s_i\rangle, \langle s_{i+1}\rangle) \wedge \bigvee_{i=0}^{k} \left(P(\langle s_i\rangle) \wedge \bigwedge_{j=i}^{k} \neg C(\langle s_j\rangle) \wedge \bigvee_{l=i}^{k} T(\langle s_k\rangle, \langle s_l\rangle) \right)$$

Transition System $\mathcal{T}(P)$ from Program P

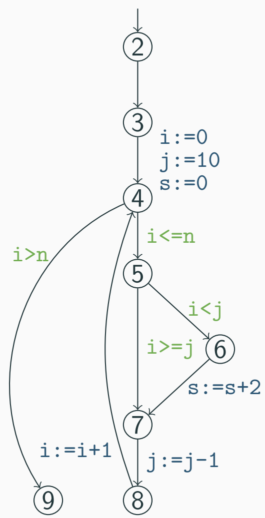
- state $\langle pc, v_0, \dots, v_n\rangle$ consists of
  - value for program counter pc, i.e. line number in P
  - assignment for variables in scope $\mathtt{v}_0, \dots, \mathtt{v}_n$
- there is step $s \to s'$ for $s = \langle pc, v_0, \dots, v_n\rangle$ and $s' = \langle pc', v_0', \dots, v_n'\rangle$ iff P admits step from s to s'
- S_0 consists of initial program states
- atom set A consists of all propositional formulas over $pc, v_0, \dots, v_n$
- labeling $L(s)$ is set of all atoms A which hold in $s = \langle pc, v_0, \dots, v_n\rangle$

Program Graph

- nodes are line numbers
- edge from line l to line l' if program counter can go from line l to l'
- two kinds of edge labels:
  - conditions for program counter to take this path
  - assignments of updated variables
- program graph is useful to derive encoding of $\mathcal{T}(P)$

Checking an Explicit Assertion

```
1  int n;
2  int main() {
3    int i, j=10, s=0;
4    for(i=0; i<=n; i++) {
5      if (i<j)
6        s = s + 2;
7      j--;
8    }
9    assert(s==n*2 || s == 0);
10 }
```



- construct program graph
- states are of form $\langle \mathrm{pc}, i, j, n, s\rangle$
- safety property to check is $\mathsf{G}\,(\mathrm{pc} = 9 \to (s = 2n \vee s = 0))$
- see `verification.py`

Software Verification Competition (SV-COMP)

- annual competition
  `https://sv-comp.sosy-lab.org/2018/`
- industrial (and crafted) benchmarks
  `https://github.com/sosy-lab/sv-benchmarks`
- many tools use SMT solvers

Common Safety Properties

- no overflow in addition: $(\mathtt{x} > 0 \wedge \mathtt{x} + \mathtt{y} \geqslant \mathtt{y}) \vee (\mathtt{x} \leqslant 0 \wedge \mathtt{x} + \mathtt{y} \leqslant \mathtt{y})$
- array accesses in bounds: $0 \leqslant \mathtt{i} < \mathit{size}(\mathtt{a})$ for all accesses a[i]
- memory safety: set predicate $\mathit{ok}(\mathit{addr})$ when memory allocated, check $\mathit{ok}(\mathtt{p})$ for every dereference $*\mathtt{p}$
- explicit assertions

Bibliography

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