1 Consider the formula

$$(\overline{1} \vee 3) \wedge (1 \vee 2) \wedge (2 \vee \overline{4}) \wedge (4 \vee 1) \wedge (\overline{3} \vee 2) \wedge (4 \vee \overline{2} \vee \overline{3})$$

Give a DPLL inference sequence to determine its satisfiability. Indicate which rule was applied in each step.

2 Consider the formula

$$(\overline{7} \vee 6) \wedge (\overline{6} \vee 5) \wedge (\overline{6} \vee \overline{5} \vee \overline{4}) \wedge (\overline{2} \vee 1) \wedge (\overline{8} \vee \overline{7}) \wedge (\overline{5} \vee 3) \wedge (\overline{3} \vee \overline{2} \vee \overline{1} \vee 4)$$

and suppose a DPLL inference sequence reached the state $7^d\, 6\, 5\, \overline{4}\, 3\, \overline{8}\, 2^d\, 1$. Construct a conflict graph and indicate three different cuts together with the induced backjump clauses. Indicate at least one UIP.

3 Find an MUC of the following formula:

$$(x \vee y \vee z) \wedge \neg x \wedge (x \vee y) \wedge (x \vee \neg y \vee z) \wedge (x \vee z) \wedge (\neg x \vee \neg y \vee z) \wedge \neg z$$

4 Consider the following formula ϕ_3:

$$(\mathsf{f}(\mathsf{a}) \approx \mathsf{b} \vee \mathsf{a} \approx \mathsf{b}) \wedge (\mathsf{f}(\mathsf{b}) \approx \mathsf{b} \vee \mathsf{a} \approx \mathsf{c}) \wedge \mathsf{f}(\mathsf{a}) \not\approx \mathsf{f}(\mathsf{b}) \wedge (\mathsf{f}(\mathsf{a}) \not\approx \mathsf{f}(\mathsf{c}) \vee \mathsf{a} \not\approx \mathsf{b})$$

(a) Construct a propositional skeleton of ϕ_3.

(b) Check satisfiability of ϕ_3 using DPLL(T).

5 Consider the following set of equations E:

$$\begin{array}{lll} \mathsf{a} \approx \mathsf{b} & \mathsf{f}(\mathsf{a}) \approx \mathsf{a} & \mathsf{f}(\mathsf{b}) \approx \mathsf{c} \\ \mathsf{g}(\mathsf{b}) \approx \mathsf{g}(\mathsf{g}(\mathsf{a})) & \mathsf{f}(\mathsf{a}) \approx \mathsf{g}(\mathsf{a}) & \mathsf{f}(\mathsf{c}) \approx \mathsf{f}(\mathsf{g}(\mathsf{c})) \end{array}$$

and use congruence closure to determine whether the following hold:
(a) $E \vDash \mathsf{f}(\mathsf{b}) \approx \mathsf{c}$ (b) $E \vDash \mathsf{f}(\mathsf{c}) \approx \mathsf{c}$

6 Suppose a run of the Simplex algorithm reached the following intermediate state:

$$\begin{array}{c} \\ s_1 \\ s_2 \\ y \\ s_4 \end{array} \begin{array}{c} \begin{array}{cc} x & s_3 \end{array} \\ \begin{pmatrix} -1 & -1 \\ -4 & -1 \\ -1 & -1 \\ 3 & 1 \end{pmatrix} \end{array} \quad \begin{array}{l} \\ s_1 \leq 4 \\ s_2 \leq -1 \\ s_3 \leq -5 \\ s_4 \leq 3 \end{array}$$

with assignment $s_3 = -5$, $x = 0$, $s_1 = 15$, $s_2 = 5$, $y = 5$, $s_4 = -5$.

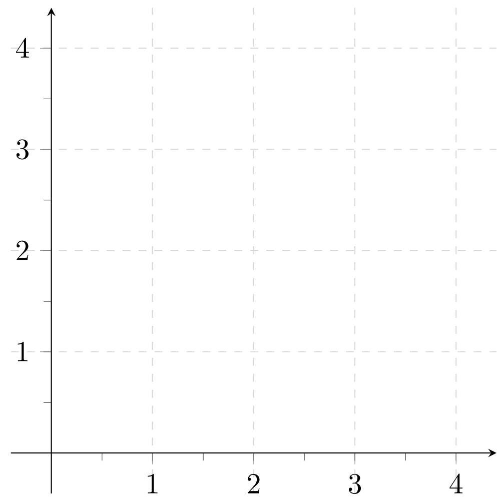
(a) Which variable pairs are suitable for a pivot step, and why?

(b) Perform one possible pivot step.

7 Consider the following system of linear inequalities:

$$2y - x \leq 2 \qquad\qquad 2y + x \geq 2 \qquad\qquad y + 2x \leq 4$$

Draw the constraints in the following diagram. How many solutions are there in $\mathbb{Z}^2$?



8 Use equality graphs to determine whether the following formula is satisfiable.

$$x_1 \approx x_2 \wedge x_2 \approx x_3 \wedge x_3 \approx x_4 \wedge x_4 \not\approx x_6 \wedge x_1 \not\approx x_5 \wedge x_6 \not\approx x_1 \wedge x_4 \approx x_9 \wedge$$
$$x_4 \approx x_1 \wedge x_1 \not\approx x_9 \wedge x_6 \approx x_7 \wedge x_7 \approx x_8 \wedge x_5 \not\approx x_8 \wedge x_2 \approx x_9$$

9 Consider three bitvector variables $\mathbf{a}_3$, $\mathbf{b}_3$, and $\mathbf{c}_3$ of three bits each.

(a) Give a propositional formula which encodes $\mathbf{a}_3 + \mathbf{b}_3 = \mathbf{c}_3$.

(b) Give a propositional formula which encodes that the addition $\mathbf{a}_3 + \mathbf{b}_3$ overflows.

10 Use the Nelson-Oppen procedure to determine satisfiability of the following formula combining uninterpreted functions and linear real arithmetic. You can use either the deterministic or the nondeterministic version.

$$z = 0 \wedge x = y + z \wedge \mathsf{f}(y) = 1 \wedge \mathsf{f}(x) = 2$$

11 Consider the following C program P:

```
int main() {
  int i = 2;
  int m = 1;
  while (i<=10) {
      m = m * 2;
  }
}
```

(a) Draw the program graph.

(b) Describe an encoding of a state of P.

(c) Define a predicate that captures the transition relation of P (for your encoding of a state).