Exercises 1 March 15, 2019

[2] 1 Consider the formula

$$(1 \vee \neg 3) \wedge (\neg 1 \vee \neg 4) \wedge (\neg 1 \vee \neg 3 \vee 4) \wedge (2 \vee 3 \vee 4) \wedge (\neg 2 \vee 4) \wedge (3 \vee \neg 4) \wedge (\neg 1 \vee 2 \vee 5)$$

Give a DPLL inference sequence to determine its satisfiability.

[3] 2 Transform the formula $\phi = \neg(p \vee \neg(q \vee r)) \wedge (q \vee r)$ to CNF using (a) Tseitin's transformation and (b) the transformation by Plaisted and Greenbaum. Give a DPLL inference sequence of one of the formulas to determine its satisfiability.

3 Three kids are talking about their ages. Adam says "If Carol is not the youngest, then I am.", and Brian claims "If I am not the youngest, then neither is Adam."

[1] (a) Assuming they all say the truth, who is the youngest of them? Encode the problem as a SAT formula using suitable variables, and use Minisat to determine its satisfiability.

[1] (b) Is there only one possible solution? Based on your result for (a), build a second formula that can be used to decide whether there exists a different solution. Again, use Minisat to solve the formula.

[3] 4 Encode the following Minesweeper board as a SAT problem and solve it using Minisat or `z3py`.

	2		1
	3		3
			2

[3] ⋆ 5 Four guys (A,B,C,D) meet at the lobby of a hotel. There are some communication problems when they are trying to make conversation.

- Among English, French, German and Italian, each of them speaks exactly two languages.
- They cannot find a language that everyone speaks, and there is only one language that three of them speak.
- Nobody understands both French and German.

- A does not speak English, but B and C need him as an interpreter.
- C speaks German, D does not, but they can communicate directly.
- A, B and D want to talk, but cannot find a language they all can speak.

Who speaks which languages? Use a SAT encoding to find it out. You can use the Z3 bindings for Python, or write a DIMACS file and check with Minisat.

Hint: It may be useful to use 16 propositional variables X_y where $X \in \{A, B, C, D\}$ and $y \in \{\text{English}, \text{French}, \text{German}, \text{Italian}\}$ such that e.g. A_{English} is true if and only if A speaks English.

Exercises marked with a $\star$ are optional. Solving them gives bonus points if you submit them before the course via OLAT or email.