

1 Consider the formula

$$\begin{aligned} & (\neg 1 \vee 2) \wedge (\neg 1 \vee \neg 2 \vee 3) \wedge (\neg 1 \vee \neg 2 \vee \neg 3 \vee 4) \wedge (\neg 2 \vee \neg 5) \wedge (\neg 3 \vee 5 \vee \neg 6) \wedge \\ & (\neg 7 \vee 8) \wedge (\neg 8 \vee 9) \wedge (\neg 8 \vee \neg 9 \vee \neg 10) \wedge (\neg 12 \vee 13) \wedge (10 \vee \neg 12 \vee \neg 13 \vee \neg 14) \wedge \\ & (\neg 8 \vee \neg 12 \vee \neg 13 \vee 14) \end{aligned}$$

and suppose a DPLL inference sequence reached the state  $1^d 2 3 4 5 \bar{6} \bar{7} 8 9 \bar{10} 11^d 12^d 13 \bar{14}$ .

- [2] (a) Construct a conflict graph. Show all cuts which correspond to possible backjump clauses.
- [2] (b) Use the approach given on the slides of Week 2 to determine possible backjump clauses by resolution.
- [2] 2 (a) Find an example for a backjump step  $Ml^dN \parallel F, C \implies Ml' \parallel F, C$  which jumps over at least 3 decision variables (so the decision levels of  $Ml^dN$  and  $Ml'$  differ by at least 3).
- [1] (b) Can we simulate the backjump step to  $Ml' \parallel F$  by multiple steps using the other inferences rules? If yes, how?
- [3] 3 George the Gorgeous Gardener wants to plant onions, peppers, broccoli, peas, potatoes, tomatoes, and garlic in his field. The vegetables should be arranged in rows, e.g. like that:



But he recently read in a book about organic gardening that one should not put “bad companion” plants next to each other: they may take away nutrients from each other, or attract insects that can harm the neighbor. According to that

- (a) neither peas nor broccoli should be planted next to onions,
- (b) peppers should not be next to tomatoes or potatoes,
- (c) potatoes should not be next to peas, onions, or tomatoes, and
- (d) garlic should not be next to peas.

So the layout drawn above is not good because broccoli is next to onions. Encode the problem into a SAT formula to determine whether there is a layout which satisfies all conditions.

*Hint:* One possibility is to use 49 variables  $x_{r,\text{veg}}$  where  $r \in \{1, \dots, 7\}$  and  $v \in \{\text{onions}, \dots, \text{garlic}\}$ , indicating that vegetable  $v$  is planted in row  $r$ .