SAT and SMT Solving | SS 2018 | LVA 703048

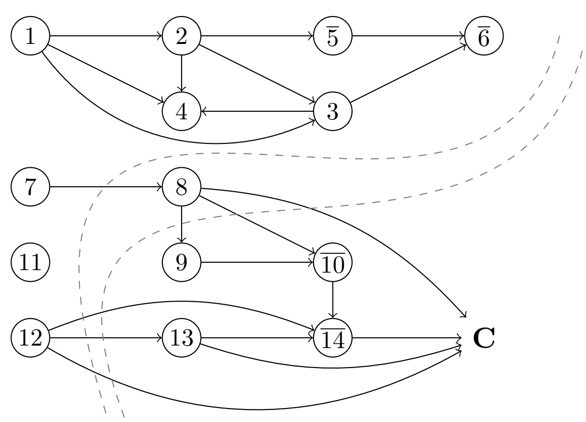
Solutions 2 | March 22, 2018

1 We consider the formula φ:

$$(\neg 1 \vee 2) \wedge (\neg 1 \vee \neg 2 \vee 3) \wedge (\neg 1 \vee \neg 2 \vee \neg 3 \vee 4) \wedge (\neg 2 \vee \neg 5) \wedge (\neg 3 \vee 5 \vee \neg 6) \wedge$$
$$(\neg 7 \vee 8) \wedge (\neg 8 \vee 9) \wedge (\neg 8 \vee \neg 9 \vee \neg 10) \wedge (\neg 12 \vee 13) \wedge (10 \vee \neg 12 \vee \neg 13 \vee \neg 14) \wedge$$
$$(\neg 8 \vee \neg 12 \vee \neg 13 \vee 14)$$

and a DPLL inference sequence reaching state $1^d\ 2\ 3\ 4\ \overline{5}\ \overline{6}\ 7^d\ 8\ 9\ \overline{10}\ 11^d\ 12^d\ 13\ \overline{14}$.

(a) The implication graph looks as follows:



The single unique implication point is node 12. The two indicated cuts lead to the implied clause $\neg 7 \vee \neg 12$ and $\neg 8 \vee \neg 12$. These are the smallest implied clauses obtained from cuts in this graph.

(b) The conflict clause is $\neg 8 \vee \neg 12 \vee \neg 13 \vee 14$, its literal whose complement was assigned last is 14. We thus resolve

$$\frac{\neg 8 \vee \neg 12 \vee \neg 13 \vee 14 \qquad 10 \vee \neg 12 \vee \neg 13 \vee \neg 14}{\neg 8 \vee 10 \vee \neg 12 \vee \neg 13}$$

The literal in the resulting clause whose complement was assigned last is 13. We hence get

$$\frac{\neg 8 \vee 10 \vee \neg 12 \vee \neg 13 \qquad \neg 12 \vee 13}{\neg 8 \vee 10 \vee \neg 12}$$

The last-assigned literal is now 12, a decision literal. But we can keep resolving with the clause that led to the assignment of the last non-decision literal, namely 10:

$$\frac{\neg 8 \vee 10 \vee \neg 12 \qquad \neg 8 \vee \neg 9 \vee \neg 10}{\neg 8 \vee \neg 9 \vee \neg 12}$$

Now the last assigned non-decision literal is 9, so obtain a next resolution step

$$\frac{\neg 8 \vee \neg 9 \vee \neg 12 \qquad \neg 8 \vee 9}{\neg 8 \vee \neg 12}$$

We proceed with a resolution step eliminating 8:

$$\frac{\neg 8 \vee \neg 12 \qquad \neg 7 \vee 8}{\neg 7 \vee \neg 12}$$

At this point there are only decision literals left, so no further steps are possible.

2 Consider the following formula φ:

$$(1 \vee \neg 2) \wedge (3 \vee 4) \wedge (4 \vee 5 \vee 6) \wedge (2 \vee \neg 3)$$

the following DPLL inference sequence:

$$\begin{aligned} \parallel \varphi \quad &\Longrightarrow^* \quad \overline{1}^d 3^d 4^d 5^d 6^d \parallel \varphi && \textsf{decide} \\ &\Longrightarrow \quad 1^d\, \overline{3} \parallel \varphi && \textsf{backjump} \end{aligned}$$

where the backjump steps uses the backjump clause $1 \vee \overline{3}$, which is entailed by the formula because it follows from resolution of the first and the last clause. It can be simulated by four backtrack steps.

3 See the file 02_garden.py.