SAT and SMT Solving | SS 2019 | LVA 703048

Selected Solutions 4 | April 5, 2019

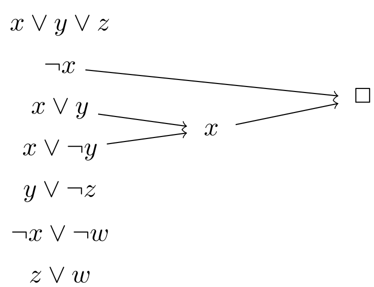
1 The following clause sets are minimal unsatisfiable cores:

(1) $\neg x,\ x \vee \neg z,\ z$

(2) $\neg x,\ x \vee y,\ x \vee \neg y \vee \neg z,\ z$

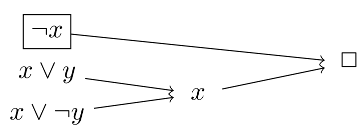
The first one is the (in this case unique) SUC.

2 We start by building a resolution graph for the given (unsatisfiable) formula. This graph is not unique, one possibility is the following:



(1) We start by picking the clause $C_1 = \neg x$. The clause set without C_1 is satisfiable, so we mark C_1.

(2) Suppose next pick $C_2 = x \vee y \vee z$. The clause set without C_2 is still unsatisfiable, so we have to build a new resolution graph for $\overline{Reach_G(C_2)}$. But since C_2 does not contribute to the current graph, we can take the same graph for $\overline{Reach_G(C_2)}$. In the last step of the algorithm the clause set is pruned to those which have a path to $\Box$:



(3) We next pick $C_3 = x \vee y$. The clause set without C_3 is satisfiable, so we mark C_3.

(4) Similarly, when picking $C_4 = x \vee \neg y$ the clause set becomes satisfiable, so we mark C_4.

At this point the algorithm terminates since all the remaining clauses C_1, C_3, and C_4 are marked. So this set constitutes a minimal unsatisfiable core (and actually a SUC).

However, a different clause selection sequence could have led to the minimal unsatisfiable core $\neg x$, $x \vee y \vee z$, $x \vee \neg y, x \vee \neg z$ (which is not an SUC).

3 (a) A 2-coloring of Austria does not exist, because the states Salzburg, Upper Austria, and Styria all share borders with each other. So a minimal unsatisfiable core will consist of the constraints that correspond to the coloring of these three states (provided the constraints are asserted as separate constraints for every state).

See for example the file austria2.py.

(b) A 3-coloring exists, see the file austria3.py.