[3] 1 Consider the following set of equations E:

$$\mathsf{a} \approx \mathsf{b} \qquad \mathsf{f}(\mathsf{a}) \approx \mathsf{b} \qquad \mathsf{f}(\mathsf{b}) \approx \mathsf{c}$$
$$\mathsf{g}(\mathsf{a}) \approx \mathsf{g}(\mathsf{g}(\mathsf{b})) \qquad \mathsf{f}(\mathsf{a}) \approx \mathsf{g}(\mathsf{b}) \qquad \mathsf{f}(\mathsf{c}) \approx \mathsf{f}(\mathsf{g}(\mathsf{c}))$$

and use congruence closure to determine whether the following hold:
(a) $E \vDash \mathsf{a} \approx \mathsf{c}$ (b) $E \vDash \mathsf{f}(\mathsf{c}) \approx \mathsf{c}$

[3] 2 Determine satisfiability of the following formula:

$$(\mathsf{g}(x_3) \neq \mathsf{g}(x_4) \vee \mathsf{g}(x_1) = x_1) \wedge (x_3 = x_4 \vee x_1 = x_2) \wedge (\mathsf{f}(x_1, x_3) \neq x_1 \rightarrow x_1 = \mathsf{f}(x_1, x_4)) \wedge$$
$$x_1 \neq x_2 \wedge (\mathsf{g}(x_3) \neq \mathsf{g}(x_4) \vee \mathsf{g}(\mathsf{f}(x_1, x_3)) \neq \mathsf{g}(x_1))$$

by transforming the formula to CNF, applying DPLL(T), and using congruence closure to check T-consistency of models.

[3] ⋆3 In a 3×3 magic square the numbers $1 - 9$ are arranged in such a manner that all rows and all columns have the same sum. Encode such a magic square in an SMT formula (in SMT-LIB, or using the `python` interface).
Which of the following two can be completed to a magical square?

	1	
4		

7		
		8

[4] 4 Solve the Rabbit puzzle by means of a SAT/SMT encoding (some integer variables might be useful):

(a) There are five houses.
(b) The Englishman lives in the red house.
(c) The Spaniard owns the dog.
(d) Coffee is drunk in the green house.
(e) The Ukrainian drinks tea.
(f) The green house is immediately to the right of the ivory house.
(g) The Old Gold smoker owns snails.
(h) Kools are smoked in the yellow house.
(i) Milk is drunk in the middle house.
(j) The Norwegian lives in the first house.

(k) The man who smokes Chesterfields lives in the house next to the man with the fox.

(l) Kools are smoked in the house next to the house where the horse is kept.

(m) The Lucky Strike smoker drinks orange juice.

(n) The Japanese smokes Parliaments.

(o) The Norwegian lives next to the blue house.

Now, who drinks water? Who owns the rabbit?

(cf. `https://en.wikipedia.org/wiki/Zebra_Puzzle`)