[3] 1 Use the Simplex algorithm to find a solution to the following system of linear inequalities in general form:

$$\begin{aligned} -x + y &= s_1 & s_1 &\leq 1 \\ -2x - y &= s_2 & s_2 &\leq -4 \end{aligned}$$

2 Consider the following system of linear inequalities:

$$\begin{aligned} y &\leq 4 \\ -3x + y &\leq -1 \\ -x - y &\leq -5 \\ 2x - y &\leq 3 \end{aligned}$$

[1] (a) Draw the solution space.

[2] (b) Use the Simplex algorithm to find a solution.

[4] 3 Solve the following instance of travelling salesman. The file `distances.py`, lists distances between 13 US cities in miles. Is there a tour (a circular route) to visit them all below 9000 miles?

The following steps might be helpful:

(a) Create 13 integer variables $c_1, \ldots, c_{13}$ with the semantics that the route is $c_1 \to c_2 \to \ldots c_{13} \to c_1$, and $c_i = 1$ iff c_i is the first city in the list (New York), $c_i = 2$ iff c_i is the second city in the list (Los Angeles), etc.

(b) Formulate a constraint that the values of all cities are between 1 and 13.

(c) Add a constraint that the values of all cities are different.

(d) Write a function `distance(c_i, c_j)` which takes two city variables and returns and expression for the distance between city c_i and c_j. You can construct this expression as a big if-then-else expression, covering all 13×13 possibilities, looking up distances in the matrix from `distances.py`.

(e) Compute an expression for the total distance of the route by summing up `distance(`c_1`, `c_2`)`, ... `distance(`c_{12}`, `c_{13}`)`, `distance(`c_{13}`, `c_1`)`.

(f) Add a constraint demanding that the total distance is below the given bound.

[4] ⋆ [4] Implement a Simplex solver for a set of linear inequalities of the form $A\vec{x} \leqslant \vec{b}$.

(This might admittedly be tricky. Details of the DPLL(T) Simplex algorithm can be found in the following technical report: Bruno Dutertre and Leonardo de Moura: *Integrating Simplex with DPLL(T)*, Technical Report SRI–CSL–06–01, SRI International, 2006.)